

# **Essays on the Financial Sector Inefficiencies**

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# ABSTRACT

## **Essays on the Financial Sector Inefficiencies**

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This dissertation investigates the sources of inefficiencies in financial sector and effects of these inefficiencies on the economy.

In the first chapter, I analyze the effects of asset prices on financial institutions in a dynamic stochastic general equilibrium model including bank defaults and related agency costs. I find that pecuniary externalities exist in asset prices as decentralized banks do not internalize the effects of their lending on asset price distributions. These externalities lead to excess risk taking and leverage in the financial sector. Excess risk taking behavior deteriorates welfare of both depositors and banks in a stochastic economy. I show that a restricted social planner is able to improve welfare by limiting the leverage in the economy. In planner's problem, robust banking system is more resilient against the shocks. This in turn creates more stable economy with lower bankruptcy costs and increases welfare. Thus, I show that significant economic gains are possible with appropriate regulations in the financial sector.

In the second chapter, I examine the welfare effects of pecuniary asset price externalities using a dynamic stochastic general equilibrium model. I show that decentralized financial system

is socially inefficient due to pecuniary price externalities. I compare various regulations using quantitative welfare analysis. I find that bailout policies cause moral hazard problems and induce excess risk taking. Therefore, such policies worsen the inefficiency. However, macro-prudential policies limit the leverage and provide resilience against the systemic shocks. Thus, these policies mitigate distortions and improve welfare. Furthermore, I show that combination of bailout and prudential reserve requirement policies is pareto better than other regulations. Finally, I introduce credit default swaps (CDS) into the model and find that CDSs can mitigate the distortions. But the benefits of CDSs are limited to the size of systemic shocks. If systemic shocks are big enough, CDS linkages will make crisis contagious among the financial institutions.

In the third chapter, I analyze the impacts of asymmetric information and imperfect monitoring on financial sector using a single period model with agency costs. I solve the model analytically comparing different levels of imperfect monitoring on heterogeneous banks. I find that information asymmetries and noises in monitoring encourage risk taking behaviors among the banks with low loan returns. I also show that these asymmetries cause inefficiently low lending among banks with high loan returns. In the extension of the model, I analyze government's incentive to prevent asymmetric information using regulatory tools such as stress tests. I analytically show that if the government is elected for short term and the rate of low return banks is high in the economy, government won't have incentive to announce real type of the banks.



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# **Chapter 1**

## **The Asset Price Externalities in Financial Intermediaries**

### **1.1 Introduction**

Financial intermediaries such as commercial banks, investment banks, hedge funds, etc. are crucial institutions in the effort to decrease inefficiency and risk in economy. The financial crisis in 2008, exacerbated by the excessive risk-taking and overborrowing of banks and their subsequent bailouts stressed the importance of the banks' structure. In their paper of 1958, Franco Modigliani and Merton Miller suggested that financial structure doesn't matter. The M-M theorem ignored bankruptcy costs and the impact of leverage on default probabilities. Banks are

fragile institutions due to their liquidity requirements at all times. The 2008 financial crisis showed that both allocation of assets and their pricing in financial institutions' balance sheets are crucial to managing risk. If banks hold similar asset types, sudden and unexpected fall of prices cause illiquidity and systemic risk among high leveraged banks. Contagious effects of asset price changes might disrupt entire financial system as in the crisis of 2008. Therefore, to prevent possible future financial crises, it is essential to re-examine the effects of asset prices on banks' risk taking decisions and their balance sheets by considering the relationships within the whole economy.

This paper presents analysis of how asset prices affect banks' leverage level and default risk in a dynamic stochastic general equilibrium (DSGE) model. I show that under competitive markets pecuniary asset price externalities lead individual banks to high risk levels and over-borrowing at social level. The main contribution of the paper is to show the existence of such pecuniary externalities in a model allowing banks' defaults with endogenous borrowing limits and to provide quantitative assessments for this model.

This paper develops an infinite period model with incomplete financial markets. The key features of the model are endogenous borrowing limits, existence of bank defaults and related agency costs.<sup>1</sup> Banks can borrow from depositors with standard debt contract using their initial

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<sup>1</sup> Literature on economic models including financial intermediaries is started with finite period models and the focus was on the relationship between lender and entrepreneur. Townsend (1979) introduced costly monitoring problem. Then literature separated into two different paths. First branch (Stiglitz and Weiss (1981) and Gale and Hellwig (1985)) concentrated on credit rationing in equilibrium while the other one tried to explain role and existence of financial intermediaries (Boyd&Prescott (1985), Diamond (1984) and Williamson (1986)). Diamond (1984) was the first study explaining the existence of banks, defining banks as delegated monitors. In Diamond

capital as partial collateral. Unlike the existing literature of pecuniary externalities, partial collateralization mechanism imposes endogenous borrowing limits and allows default of banks in the equilibrium. Banks give loans to capital good firms with perfect state contingent contract or in other words equity debt. Capital good firms sell their capital to consumption good firms in a competitive market and transfer their revenues to the banks. At the end of the period, banks' shareholders decide how much of the banks' profit will be used as dividend and as next period's initial capital.

In the model, banks have access only to non-contingent debt contracts in deposit markets. Borrowing rates are ex-ante endogenously determined in the markets with respect to supply and demand for deposits and banks' leverage levels. If banks' leverage is high, markets will ask higher interest rate to insure against the excess default risk. In this framework, banks will default with limited commitment if their revenues are not adequate to pay their debt at the end of the period. In case of default, there is costly state verification (Townsend ,1979) and some of banks' revenues are lost as agency cost.

At the end of each period, banks ex-ante decide on the next period's lending and leverage levels. While giving their decisions, banks are exposed to both idiosyncratic and systemic risks. The source of idiosyncratic risks is the uncertainty in capital good firms' productivities and such risks create heterogeneity among the banks. On the other hand the cause of systemic risk is the

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(1984) monitoring decision was ex-ante, while in Townsend (1979), Gale and Hellwig (1985) and Williamson (1986) agents are informed asymmetrically ex-post on the realization from projects and monitoring decisions were ex-post.

uncertainty in capital good prices and it affects all banks' portfolios. Banks are fully rational and they correctly perceive these risks associated with their lending decisions. Nevertheless atomistic banks do not internalize their contribution to asset prices and impose pecuniary externality on each other. This prevents atomistic banks to use the right asset price distribution in their ex-ante decisions, and induces them to take excessive systemic risk.

In the private equilibrium, I show that atomistic banks undervalue the gains from lower leverage levels by ignoring the general equilibrium effects on price distribution. I analyze the problem using a restricted social planner that faces similar constraints in private equilibrium. I find that even a restricted social planner can make all the agents better off by choosing lower leverage levels. I show that decline in lending and capital goods supply raises capital goods prices. It improves marginal profitability of the banks and thus their default risk becomes lower than their expectations. Furthermore since banks are less risky, spreads in their borrowing rates decline and their profitability increases more. Therefore inefficiency in the economy decreases as it suffers less from bankruptcy costs. On the other hand, risk averse depositors are better off as their income stream becomes less volatile with more robust banking system against shocks. Therefore a restricted planner is able to mitigate the distortions in the decentralized equilibrium and achieve a pareto efficient allocation.

The model in this paper builds on the literature of macroeconomics models with financial frictions described by Bernanke and Gertler (1989), Bernanke, Gertler and Gilchrist (1999) and Carlstrom and Fuerst (1997). These papers and following work in the literature introduced agency costs and banking sector into dynamic macroeconomic models to amplify the

macroeconomic shocks.<sup>2</sup> They compared the result with the first best solution where frictions don't exist. This paper converts the single period problem in Yildiz (2011) into the general equilibrium model using the similar approach in Carlstrom and Fuerst (1997). In order to focus on relations between asset prices and banking sector, I eliminate the monetary authority and all other nominal frictions (price frictions) from the model. Different than the literature, I use second order approximation which could be crucial in welfare comparisons. Furthermore, I compare the competitive equilibrium with social planner's solution, in which financial constraints in private equilibrium still exist (second best), instead of first best benchmark.

The paper is also related to the literature on generic inefficiency of competitive equilibrium under incomplete markets (Hart, 1975; Stiglitz, 1982; Geanakoplos and Polemarchakis, 1986). The more recent contributions in this literature (Lorenzoni, 2008; Korinek, 2011; Jeanne and Korinek, 2011; Bianchi, 2010) analyzed the role of pecuniary externalities in the inefficiencies of decentralized equilibrium under financial frictions. Lorenzoni (2008) and Korinek (2011) used finite period model in which entrepreneurs borrow in the first period and face the risk of binding financial constraints in the following period. Entrepreneurs' defaults are excluded by allowing

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<sup>2</sup> Hellmann, Murdock and Stiglitz (2000) mentioned how competitive environment can increase risk taking behavior. Greenwald and Stiglitz (2003) explained in their book possible ways of setting up general equilibrium mechanism with a banking system. However, until the 2008 financial crisis, importance of intermediaries has been largely underestimated and they were modeled simply as transmission channels of credits. After the crisis, recent studies added financial intermediaries into the DSGE models more in details (Iacoviello (2005), Goodfriend & McCallum (2007), Van den Heuvel (2008), Curdia and Woodford (2009), Christiano (2010), Dib (2010) and Gerali (2010)). All these papers examined the effect of bank values on risk taking behavior in a dynamic model but the loan and deposit markets were not micro founded. Gertler and Karadi (2011) diverted from previous DSGE models and defined long term banks. But bank runs were just threat and banks' default rate were exogenous.

the fire sales of productive assets. In financially binding states, fire sales cause pecuniary externalities in asset prices and this is ex-ante undervalued by atomistic entrepreneurs. Pecuniary externalities exist only in fire sales of productive assets because entrepreneurs are not allowed to sell their production in the markets. In this paper, by contrast, all production goods can be sold in the markets, and furthermore, banks will default if they can't pay their debt. Thus, I am able to analyze effects of pecuniary externalities in all states, and moreover, I show that generic inefficiencies of decentralized equilibrium still exist without amplification effects of fire sales in binding states.

Bianchi (2010) and Jeanne and Korinek (2011) analyzed similar pecuniary externalities using open economy DSGE models. In these papers, insiders borrow from outsiders and their default is excluded by full collateralization. Borrowing limits depend on asset values with exogenous parameters. Borrowing limits are binding only in bad states and amplify the effects of crisis by depressing the asset prices more. Atomistic agents overborrow by ignoring pecuniary externalities and this is the source of inefficiency in the economy. In these studies default is not allowed and pecuniary externality exists only in financially constrained states. Furthermore, outsiders' utilities are excluded by open economy assumption in welfare analysis. In this paper, by contrast, there is partial collateralization in a closed economy and borrowing limits are endogenously determined in the market. Thus, I can analyze bank defaults and consider the welfare of all agents in the model. Moreover, in this study, pecuniary externalities exist in all states of the economy.

The rest of the paper is organized as follows. Section 2 introduces the benchmark model. Section 3 characterizes the competitive equilibrium. Section 4 analyzes the social planner's problem. Section 5 presents the quantitative analysis including sensitivity analysis and section 6 concludes.

## 1.2 The Model

The model is a closed economy DSGE model and it consists of continuum of agents with unit mass. Basic structure of the model as follows: There are two types of atomistic agents; depositors (fraction  $1 - \eta$ ) and banks' shareholders (fraction  $\eta$ ). Depositors are the identical households living forever. They work, save and consume. Shareholders are the owners of the banks and each shareholder has shares in a specific bank. Shareholders have two sources of income; labor income and profit of the banks. They can either consume or save by investing in banks as capital. Both shareholders and banks are heterogeneous in terms of their initial capital.<sup>3</sup> Banks' managements are different than shareholders. They have short term, one period, contract and their objective, given the initial capital, is to maximize banks' profits at the end of each period. At the end of each period, banks borrow from depositors to invest in next period's capital good production. While borrowing, banks' managements solve the financial contract problem. Since financial contract is just one period in length, it is explained separately from general

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<sup>3</sup>Keeping track of variables' mean instead of their distribution is adequate due to the assumption of atomistic agents and law of large numbers.



equilibrium model. There are two types of firms in the model; capital and consumer good firms. Capital good firms generate demand for bank's loans. On the other hand, consumer good firms provide wage income for agents and generate demand for capital goods.

### 1.2.1 Financial Contract

Financial contract is the standard debt contract between banks and depositors with limited commitment of the banks and it limits the banks' borrowing endogenously (Williamson, 1986; Carlstrom and Fuerst, 1997). Financial contract generates a supply curve for capital goods and optimality conditions of the contract are embedded into the general equilibrium model in the following sections.

Banks have two sources for lending; deposits ( $d_t$ ) and initial capital ( $w_t$ ). Atomistic depositors are the only suppliers of deposits. Depending on the average expected return ( $a_t$ ) from each unit of deposit, depositor saves  $d_t^s$  unit of consumption good and distributes his savings among the banks. Law of large numbers allows them to insure against idiosyncratic risk of the banks' lending activities. But depositors are still exposed to the banks' default risk due to systemic risk. Depositors can observe leverage ratio and estimate the default probability of the banks. Thus, in the contract, depositors ask spread over risk free rate to insure them against default risk. Based on similar reasoning, banks face with different interest rate for each level of leverage in their optimization problem.

There is perfect competition in banking sector and banks optimize next period's expected profits by choosing the optimum level of borrowing. This generates demand for depositors'

saving. Next period, banks pay interest rate  $r^d$  when they do not default. When banks default, depositors get all the remaining capital and pay the bankruptcy cost, defined as monitoring and liquidation costs. In the deposit markets,  $r_t^d$  and  $a_t$  clear the market and equalize total saving of depositors to total deposit demand from the banks,  $(1-\eta)d_t^s = \eta d_t$ .<sup>4</sup>

Banks' revenues depend on the performance of their investment in capital good firms. Capital good firms' initial endowment is zero. In order to produce, they need to borrow  $l_t$  units of loan. There is one to one match between banks and firms. Each firm borrows only from a specific bank. Firms' technology is linear,  $n_t l_t$ .  $n_t$  is the productivity factor and randomly distributed over the interval  $[n^L, n^H]$  with the density function  $f(n_t)$ , assumed uniform. Both banks and capital good firms don't know productivity level before production. The banks have perfect monitoring skills over firms. The contract between firms and the banks is assumed perfectly state contingent debt contract similar to issuing equity for debt. Capital good firms sell capital goods to consumption good firms at price  $q_t$  and transfer all their returns to the banks.

If bank's revenues are not enough to pay its liabilities to depositors ( $q_t n_t l_t < r^d d_t$ ), it will default. Productivity parameter of capital good firms is crucial to determine default risk. There exists a threshold level of  $n$  such that any realization of productivity below this threshold causes default.

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<sup>4</sup> Deposit demand is different among the heterogeneous banks. But assumption of law of large numbers allows us using  $d_t$  (deposit demand of average bank) instead.

$$\tilde{n}_t = \frac{r_t^d d_t}{q_t(d_t + w_t)} \quad (1.1)$$

Banks' default probability is the cumulative distribution function of  $n$  at the threshold level,  $F(\tilde{n}_t)$ . Under the financial contract with given  $q_{t+1}$ , banks' expected profit (in terms of consumption good) for  $t+1$  at period  $t$  is:

$$\int_{\tilde{n}_{t+1}}^{n_H} q_{t+1} n_{t+1} l_{t+1} f(n) dn - (1 - F(\tilde{n}_{t+1})) r_{t+1}^d d_{t+1}$$

Figure 1.1 displays the profit of decentralized bank as a function of lending under benchmark calibration given in quantitative analysis section. As it is also shown in Appendix A, bank's profit is concave function of its lending. Bank's profit function can be simplified to:

$$q_{t+1} l_{t+1} \pi_{t+1}^B = q_{t+1} l_{t+1} \left( \int_{\tilde{n}_{t+1}}^{n_H} n_{t+1} f(n) dn - (1 - F(\tilde{n}_{t+1})) \tilde{n}_{t+1} \right) \quad (1.2)$$

where  $\pi_{t+1}^B$  is the bank's return from each unit of lending in terms of capital good. Similarly a bank's expected total payment to all its depositors, given  $q_{t+1}$  is:

$$\int_{n_L}^{\tilde{n}_{t+1}} q_{t+1} (1 - \mu) n_{t+1} l_{t+1} f(n) dn + (1 - F(\tilde{n}_{t+1})) r_{t+1}^d d_{t+1}$$

where  $\mu$  is the bankruptcy cost parameter. It can be rewritten as;

$$q_{t+1} l_{t+1} \pi_{t+1}^D = q_{t+1} l_{t+1} \left( \int_{n_L}^{\tilde{n}_{t+1}} (1 - \mu) n_{t+1} f(n) dn + (1 - F(\tilde{n}_{t+1})) \tilde{n}_{t+1} \right) \quad (1.3)$$

where  $\pi_{t+1}^D$  is the depositors' return from each unit of lending in terms of capital good. At the end of each period, banks optimize next period's expected profits subject to depositors'

participation constraint. The optimal contract is the solution of the bank's optimization problem given by:

$$\max_{l_{t+1}} E_t q_{t+1} l_{t+1} \pi_{t+1}^B, \text{ subject to } E_t q_{t+1} l_{t+1} \pi_{t+1}^D \geq a_{t+1} d_{t+1} \quad (1.4)$$

Depositors are identical and their participation constraint is always binding. It is assumed that returns are attractive enough to convince banks to borrow. Therefore, banks' participation constraint is always satisfied and can be ignored while solving the optimization problem. It is important to mention that shareholders own the banks but it is the bank's management who is in charge and solves the contract problem. Since managements' employment contract is short term, their goal is to maximize the next period profits and they ignore further future profits (agency problem). This induces risk taking behavior of banks as future losses in case of possible defaults are ignored in the financial contract.

### 1.2.2 General Equilibrium Model

In the general equilibrium model, infinitely-lived, identical, atomistic depositors' preference is given by:

$$U^D = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^D) \quad (1.5)$$

In this expression,  $\beta$  is the discount factor and utility of depositor has the constant-relative-risk-aversion (CRRA) form. Each period, depositors allocate their incomes between saving and consumption according to budget constraint given by:

$$d_{t+1}^s + c_t^D \leq w_t^d h_t + q_t l_t \pi_t^D \eta / (1 - \eta) \quad (1.6)$$

where  $c_t^D$  is the consumption,  $h_t$  is the labor supply and  $w_t^d$  is the real wage. Labor supply is assumed constant to focus on financial relations. The second term at the right side of the budget equation is the return from savings. Since depositors are identical it is calculated by averaging the total payments from the banks.

Banks' shareholders have two sources of income; banks' profits and wage income. At the end of each period, they decide how much of the income to consume and reinvest in banks. Their reinvestments are used as initial capital for banks' next period operations. In order to induce shareholders' consumption and thus banks' borrowing in the model, I assumed that shareholders discount future more heavily than households. Shareholders' preference is given by:

$$U^B = E_0 \sum_{t=0}^{\infty} (\beta\gamma)^t c_t^B \quad (1.7)$$

where  $c_t^B$  denotes shareholder's time  $t$  consumption and  $\gamma \in (0,1)$  denotes the additional discount rate. Shareholders supply constant labor each period and this allows them to enter the banking system next period even if they default. Shareholders' budget constraint is given by:

$$w_{t+1} + c_t^B \leq q_t l_t \pi_t^B + w_t^B \quad (1.8)$$

where  $w_{t+1}$  is bank's capital in next period and  $w_t^B$  is the shareholder's wage.

Consumption good producers buy capital from capital goods firm and hire labor from depositors and shareholders. Capital is fully depreciated at the end of each period. Their production technology is given by:

$$Y_t = \theta_t (K_t)^\alpha (H_t)^{1-\alpha-\alpha_H} (\eta)^{\alpha_H} \quad (1.9)$$

where  $\theta_t$  denotes the stochastic productivity parameter,  $K_t$  denotes the aggregate supply of capital goods,  $H_t$  denotes the aggregate supply of depositors' labor and  $\eta$  denotes the aggregate supply of shareholders' labor.

## 1.3 Equilibrium

### 1.3.1 Optimality Conditions

In the financial contract (1.4), bank's problem at time  $t$  is to choose  $l_{t+1}$  to maximize its expected profit subject to depositor's participation constraint. The optimality conditions require<sup>5</sup>:

$$q_{t+1}(\bar{n} - \mu \frac{(\tilde{n}_{t+1})^2 - (n^L)^2}{2(n^H - n^L)} + \mu \frac{\tilde{n}_{t+1}}{n^H - n^L} \frac{\pi_{t+1}^B}{\pi_{t+1}^B}) = a_{t+1} \quad (1.10)$$

$$q_{t+1} l_{t+1} \pi_{t+1}^D = a_{t+1} d_{t+1} \quad (1.11)$$

where  $\bar{n}$  is the expected return. Optimality condition (1.10) equates marginal profit of bank from an extra unit of lending to the marginal cost of borrowing. It determines the threshold level of  $n$ ,  $\tilde{n}_{t+1}$ , independent of bank's capital. Thus optimal threshold level of each bank is same. Equation (1.11) is the binding participation constraint of depositors and it implies that the interest rate is the function of bank's leverage ratio. As the threshold level of  $n$  depends on interest rate and leverage ratio in equation (1.1), optimality conditions (1.10 and 1.11) imply that

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<sup>5</sup> Detailed solution of financial contract is given in Appendix A.1.

all banks choose same leverage ratio in the financial contract.<sup>6</sup> It allows great simplification in the solution of the general equilibrium model while preserving the heterogeneity among the banks.

In the general equilibrium model, depositors choose the stochastic processes  $\{c_t^D, d_{t+1}^s\}_{t \geq 0}$  to maximize their utility (1.5) subject to their budget constraint (1.6). The optimality conditions require<sup>7</sup>:

$$u_{c,t} = \beta E_t u_{c,t+1} \left( (1-F(\tilde{n}_{t+1}))r_{t+1}^d + (1-\mu)q_{t+1} \frac{l_{t+1}}{d_{t+1}} \int_{n_L}^{\tilde{n}_{t+1}} n_{t+1} f(n) dn \right) \quad (1.12)$$

$$d_{t+1}^s + c_t^D = w_t^d h_t + q_t l_t \pi_t^D \eta / (1-\eta) \quad (1.13)$$

The optimality condition (1.12) is the euler equation. Expression in the brackets is the depositor's next period's return from extra unit of saving.

Shareholders choose the stochastic processes  $\{c_t^B, w_{t+1}\}_{t \geq 0}$  to maximize their utility (1.7) subject to the budget constraint (1.8). The optimality conditions are given by:

$$\frac{1}{\beta\gamma} = q_{t+1} \pi_{t+1}^B \frac{l_{t+1}}{w_{t+1}} \quad (1.14)$$

$$w_{t+1} + c_t^B = q_t l_t \pi_t^B + w_t^B \quad (1.15)$$

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<sup>6</sup> Detailed proof is given in Appendix A.1.

<sup>7</sup> Detailed derivations of depositors and bankers optimality conditions are given in Appendix A.

Equation (1.14) is the euler equation for the shareholders. Expression at the right side of equation is the bank's return from extra unit of capital.

There are four markets; deposit, labor, capital goods and consumption goods. Competition in the factor markets implies that wages and capital good prices are equal to their marginal products:

$$w_t^d = (1-\alpha-\alpha_H) \frac{Y_t}{H_t} \quad \text{and} \quad w_t^B = \alpha_H \frac{Y_t}{\eta_t} \quad (1.16)$$

$$q_t = \alpha \theta_t (K_t)^{\alpha-1} (H_t)^{1-\alpha-\alpha_H} (\eta)^{\alpha_H} \quad (1.17)$$

Technology parameter of the consumption good firms,  $\theta_t$ , is the source of systemic risk in the economy via capital good prices.

Labor, deposit, capital goods and consumption goods market clearing conditions are given by<sup>8</sup>:

$$H_t = (1-\eta)h_t \quad (1.18)$$

$$K_t = \eta \bar{n} l_t \quad (1.19)$$

$$(1-\eta)d_t^s = \eta d_t \quad (1.20)$$

$$Y_t = (1-\eta)c_t^D + \eta c_t^B + \eta F(\tilde{n}_t) \mu q_t l_t + \eta l_{t+1} \quad (1.21)$$

---

<sup>8</sup> Aggregate values are calculated by multiplying the fraction of agents in population with the average value of variables.



### 1.3.2 Equilibrium Definition

There are five state variables in the decentralized optimization problem. At period  $t$ , banks and depositors decide on next period's lending ( $l_{t+1}$ ) and starting capital ( $w_{t+1}$ ). In deposit markets, depositor's average expected future return ( $a_{t+1}$ ) is determined to clear the deposit market. Similarly deposit interest rate ( $r_{t+1}^d$ ) is determined to insure the depositors. The other state variable is the technology of consumption good firms. Rational agents perceive actual levels of aggregate lending and capital. They estimate asset price using equation (1.17).

A decentralized competitive equilibrium for this economy is defined by interest rate  $r_{t+1}^d$ , deposit return  $a_{t+1}$ , wages  $w_t^D$  and  $w_t^B$ , asset price  $q_t$  and decision rules for  $c_t^D$ ,  $c_t^B$ ,  $w_{t+1}$ ,  $l_{t+1}$ ,  $d_{t+1}$ ,  $K_{t+1}$ ,  $\tilde{n}_t$  where they are the functions of  $(r_t^d, a_t, l_t, w_t, \theta_t)$  and satisfy the following:

- (i) Financial contract optimality conditions (equation 1.10 and 1.11).
- (ii) Depositors' optimality conditions (equation 1.12 and 1.13).
- (iii) Shareholders' optimality conditions (equation 1.14 and 1.15).
- (iv) Market clearing conditions (equation 1.16 to 1.21).

Equilibrium of this decentralized model exists and it is unique. Detailed proof is given in Appendix B.

## 1.4 Social Planner

In the decentralized equilibrium, atomistic agents take aggregate variables as given, particularly total lending ( $L_{t+1}$ ). While solving the financial contract, individual banks don't internalize their contribution to total lending ( $dL_{t+1}/dl_{t+1}=0$ ) which implies that banks ignore the effects of their actions on the capital good's price distribution ( $dq_{t+1}/dl_{t+1}=0$ ). Thus atomistic banks overvalue the marginal gains from lending in their optimality condition (1.10). This induces atomistic banks to take excess risk by lending more, and hence high default rates are observed at the decentralized equilibrium.

### 1.4.1 Planner with Full Capability

Planner who can arbitrarily redistribute the funds between agents in the economy can implement the first best solution. Planner perceives the bankruptcy as the source of inefficiency since resources are spent on inefficient activities such as monitoring costs. As planner is capable of transferring money between agents, it can eliminate defaults and bankruptcy costs in the equilibrium. Social planner's problem with full capability is formulated as:

$$\begin{aligned}
 & \max_{\{c_t^D, c_t^B\}_0^\infty} E_0(1-\eta) \sum_{t=0}^{\infty} \beta^t u(c_t^D) \\
 & \text{subject to} \\
 & (1-\eta)c_t^D + \eta c_t^B + L_{t+1} \leq \theta_t (K_t)^\alpha (H_t)^{1-\alpha-\alpha_H} (\eta)^{\alpha_H} \\
 & K_t = \bar{n} L_t \\
 & U^B \geq (U^B)^{DE}
 \end{aligned} \tag{1.22}$$

where superscript ‘DE’ refers to decentralized equilibrium and  $(U^B)^{DE}$  represents the lifetime utility level of average shareholder in decentralized equilibrium. Solution of the planner’s problem and its optimality conditions are given in Appendix C. Simulation results given in the quantitative analysis section show that social planner’s allocation constitutes a pareto improvement both in the steady state (economy without shocks) and in the stochastic environment. Social planner chooses higher level of lending compared to decentralized equilibrium as it can prevent defaults with ex-post transfers. Therefore it proves that decentralized equilibrium is not the first best solution and economy can be potentially better off.

#### 1.4.2 Planner with Restricted Capability

In the previous section, I describe the planner’s problem with full capability in which financial contracts and agency costs are irrelevant. But, due to the excess capability of planner, it is not a suitable reference point for possible government regulations. Therefore, in the remainder of the paper, I assume planner’s abilities are limited. I suppose that planner has control only over the banking sector and it can’t make ex-post transfers to the banks and depositors<sup>9</sup>. I assume that planner has regulatory tools to control the banks’ lending in the financial contract. Thus planner can interfere only in financial contracts on behalf of the banking sector.

The social planner solves the financial contract problem (1.4) by internalizing effects of lending decisions on asset prices. The optimization problem is formulated as:

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<sup>9</sup> This assumption will be relaxed in the following paper while testing various policy recommendations.

$$\begin{aligned}
& \max_{l_{t+1}} E_t q_{t+1} l_{t+1} \pi_{t+1}^B \\
& \text{subject to} \\
& E_t q_{t+1} l_{t+1} \pi_{t+1}^D \geq a_{t+1} d_{t+1} \\
& E_t q_{t+1} = E_t \alpha \theta_{t+1} (\bar{n} \eta l_{t+1})^{\alpha-1} (H_{t+1})^{1-\alpha-\alpha_H} (\eta)^{\alpha_H} \\
& U^D \geq (U^D)^{DE}
\end{aligned} \tag{1.23}$$

where  $(U^D)^{DE}$  represents the lifetime utility level of average depositor in decentralized equilibrium. In order to have an interior solution, I assume that depositors' utility constraint ( $U^D \geq (U^D)^{DE}$ ) is not binding.<sup>10</sup> Given  $\theta_{t+1}$ , optimality condition is given by:

$$\alpha q_{t+1}^S (\bar{n} - \mu \frac{(\tilde{n}_{t+1})^2 - (n^L)^2}{2(n^H - n^L)} + \mu \frac{\tilde{n}_{t+1}}{n^H - n^L} \frac{\pi_{t+1}^B}{\pi_{t+1}^D}) = a_{t+1} \tag{1.24}$$

where  $q_{t+1}^S$  is the capital good price in social planner's equilibrium. Comparing equation (1.10) and equation (1.24), planner perceives lower marginal benefit from excess lending levels of decentralized banks. Figure 1.2 displays banks' profits from perspective of planner and atomistic banks.<sup>11</sup> Difference between two viewpoints arises from the asset price perception of the planner and atomistic banks as illustrated in Figure 1.3.<sup>12</sup> Planner internalizes its contribution on asset price distribution while atomistic banks assume that prices are independent of their lending. Thus planner chooses lower lending levels than the decentralized banks. The decline in

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<sup>10</sup> In Appendix D and in quantitative analysis section, I show that restricted planner solution is still pareto better when depositors' utility constraint is binding and there is corner solution.

<sup>11</sup> Figure 1.2 is prepared using benchmark calibration explained in quantitative analysis section.

<sup>12</sup> Figure 1.3 is prepared using benchmark calibration explained in quantitative analysis section.

banks' lending implies a decline in supply of capital goods which causes higher asset prices in planner's problem ( $q_{t+1}^S \geq q_{t+1}^{DE}$ ). Therefore, in planner's problem, even banks' lending is lower, banks' profits are higher than the decentralized equilibrium and risk neutral shareholders are always better off.

Notice that planner's solution implies transfer of revenues from depositor to bankers via asset prices and capital good supply. Low supply of capital goods decreases the wage income of depositors at the steady state where there is no aggregate uncertainty. On the other hand, having low leverage and less risky banks decreases the volatility of risk-averse depositors' incomes. In the stochastic environment, low default rates allow banks to accumulate more capital in the long run and make them more robust against shocks. This provides safer deposit income for depositors by minimizing bankruptcy costs. If the variance of price shocks in the economy is high enough, risk-averse depositors' benefits from having low leverage banks will exceed their wage losses. Thus depositors' utility constraint will not be binding and planner's solution will be an interior solution.

**Proposition 1:** *There exists a threshold level of variance of shocks,  $\tilde{\sigma}^2$ , such that when  $\sigma^2 \geq \tilde{\sigma}^2$  social planner's solution is always interior solution. Under this interior solution, all agents are pareto better and decentralized equilibrium is not constrained efficient.*

Proof: See Appendix D.

Inefficiency of the decentralized equilibrium is not restricted with variance of the shocks. Size of the variance determines only whether the planner's problem (1.23) has an interior or a

corner solution. Even if the size of shocks is low in the economy, social planner can always choose a marginal decrease in lending which makes banks better off and risk-averse depositors indifferent (corner solution) due to benefits from lower level of volatility and more robust banks. Results of proposition 1 are generalized in proposition 2.

**Proposition 2:** *Social planner can always choose an allocation which improves welfare of all agents for any level of variance of the shocks. Therefore, in general, decentralized equilibrium is not constrained inefficient.*

Proof: See Appendix D.

## 1.5 Quantitative Analysis

In this section, I describe the quantitative implications of the model. I present calibration of the model, numerical solution of decentralized problem with shocks and sensitivity analysis. While solving the model numerically, I use second order approximation with perturbation method. Since the model is set up in stochastic environment, second order approximation improves the accuracy of the model and estimations (Schmitt-Grohe and Uribe, 2004).

### 1.5.1 Calibration

The model is parameterized using standard values in the related literature. The risk aversion parameter,  $\sigma$ , is set at 2. Quarterly discount factor  $\beta$  is set at 0.96, implying 4 percent annual average return on deposits at steady state. In consumption good production, the capital share ( $\alpha$ ) is set at 0.35. Labor share of depositors and shareholders are set at 0.64 and 0.01 respectively.

The shareholders' labor share ensures that shareholders have positive net worth even after default to make a fresh start. It also guarantees the positive consumption of shareholders in case of default.

Productivity factor of capital good firm,  $n$ , is randomly distributed over the interval  $[0,2]$  with the uniform density function as in Carlstrom and Fuerst (1997). The variance of consumption good firm's technology parameter ( $\theta$ ) is set to 0.36, in keeping with the literature. Results with different levels of variance are provided in sensitivity analysis. The ratio of shareholders in population doesn't have significant effect on the results and it is set at  $\eta = 0.33$ .

As for the bankruptcy cost parameter, there is considerable controversy in the literature. Among the similar class of models, Carlstrom and Fuerst (1997) defined the reasonable interval as  $(0.2, 0.36)$  by referring to the empirical studies of Altman (1984) and Alderson and Betker (1995). Bernanke, Gertler and Gilchrist (1999) set the parameter to 0.12. All of these studies defined the bankruptcy costs for a classical firm rather than a financial institution. The bankruptcy cost is underestimated for the banking sector considering the Lehman case in 2008, the largest bankruptcy filed with \$ 691 billion in assets. According to FDIC report<sup>13</sup>, current bankruptcy plan can recover only 20 cents on every \$ 1 of claims and the optimistic completion date is 2014, 6 years after the bankruptcy filing. In the benchmark model, I set bankruptcy cost to 0.36, upper level in the literature. But in sensitivity analysis results with higher bankruptcy costs are provided.

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<sup>13</sup> 'The Orderly Liquidation of Lehman Brothers Holdings Inc. under the Dodd-Frank Act', FDIC Quarterly (2011)

The final parameter to be selected is additional discount rate of bankers ( $\delta$ ). This parameter allows shareholders to discount future more than depositors and induces the lending in the model. The focus of the paper is the leverage of the banks, so contrary to literature discount parameter is selected to match the leverage ratio of financial institutions. Leverage ratio used in Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999) is 2.5 and 0.5, respectively. Considering that average leverage of US banks varies between 20 and 40 over time (from 1992 to 2008)<sup>14</sup>, leverage ratios used in the literature are too low for financial institutions and they target classical firms instead. Since the focus of paper is financial institutions, discount rate is set to  $\delta=0.3$  which implies leverage at 22.

### 1.5.2 Response to Asset Price Shock

The main goal of this section is to analyze the response of financial sector and agents to unexpected asset price changes. In order to produce such asset price changes, productivity of consumption good firm is used. Consumption good firms are the buyers of capital goods and considering asset price formula (equation 1.17), any change in their productivity affects the asset prices directly.

The technology of the consumer good firm is given by;

$$\log\theta_{t+1} = \rho\log\theta_t + \varepsilon_t \quad (1.25)$$

---

<sup>14</sup> Average leverage levels of US banks are reported since 1992 in Adrian and Shin (2010).



Where  $\rho$  is the autocorrelation coefficient and  $\varepsilon_t$  is serially uncorrelated shock. In the benchmark calibration, shocks are assumed to be persistent,  $\rho=0.9$ .

In the simulations, I analyze response of economy to one percent decrease in productivity (one percent decrease in asset prices) at the steady state. The results are presented in Figure 1.4 and Figure 1.5. Figure 1.4 displays the response of financial sector. In the initial period, asset price declines one percent. Since current period's deposit rates, loans, banks' capital and leverage are predetermined with the financial contract in previous period, financial sector's initial response is limited. In the current period, drop in asset prices decreases the profits of the banks which cause an increase in default probability and a decline in next period's starting capital. Banks couldn't respond as they are tied with the financial contract which causes a further decline in their profitability.

In the following periods, loss of capital together with low asset prices creates upward pressure on default risks. In order to pull back default probability to steady state levels, banks cut their lending significantly, more than one percent. Extreme cut in loans causes a drop in the leverage level and decline in the capital good supply. Low supply of capital goods creates an asset price recovery and asset price exceeds the steady state levels two periods after the shock. In the deposit markets, deposit supply declines with the low wage income of depositors. However deposit demand of banks decline further and pulls deposit rates down. Deposit rates recover in line with asset prices, as demand from banks is getting better with increasing profitability. In the long run, financial variables revert to the steady state levels due to temporality of the shock.

Figure 1.5 displays the response of agents' consumption to the shock. In the initial period, drop in asset prices causes a significant decline in both depositors' and shareholders' consumption. Initial decrease in shareholders' consumption is around 5 percent. The reason for such a decline is the significant losses in the first period. Since banks' capital is crucial for future profitability and default risks, shareholders prefer to cut more from their consumption and limit the decline in banks' capital. In the following period, banks are able to optimize their leverage in financial contract with respect to the shock and they can minimize the losses. Therefore, shareholders' consumption converges back rapidly, but it is still lower than steady state. In the long run, shareholders' consumption converges to steady state levels as shocks are temporary. Initial response of depositors to the asset price shock is to cut their consumption. Decreasing wage income and deposit revenues amplify the decline in the following period. Decline in consumption is higher than one percent but it is more tolerable compared to drop in shareholders' consumption. In the long run, as wages and deposit revenues come back to steady state levels, depositors' consumption converges back to steady state levels too.

Figure 1.5 displays that negative asset price shock initiates a contraction period in the economy. Since banks cut lending in the following period, capital good supply decreases. This amplifies the contraction of the output one period after the shock. In the long run, output converges to steady state with a slow pace. These results imply the pro-cyclicality of bank's leverage with output in the economy which is in line with the literature.

### 1.5.3 Sensitivity Analysis

In this section, I examine the sensitivity of results to parameters of the model. I analyze the effects of varying each parameter on the results separately by calculating the unconditional moments of variables.

#### 1.5.3.1 Bankruptcy Cost ( $\mu$ )

Figure 1.6 displays how the model's results change with respect to bankruptcy cost. An increase in bankruptcy costs means lower revenues for depositors in case of bank's defaults. Therefore, depositors increase interest rates to insure themselves. As banks pay higher interest rate for same level of deposits, they cut their leverage. Low lending decreases capital good supply and output in the economy. Such a decline capital supply induces a cut in depositor's wage income and consumption. Therefore depositor's utility decreases with higher bankruptcy costs. On the other hand, low capital supply causes higher asset prices. Banks have limited commitment and gains from high asset prices overweight deposit rate increases. Hence, utilities of banks' shareholders increase with the bankruptcy costs.

#### 1.5.3.2 Risk Aversion ( $\sigma$ )

An increase in risk aversion means higher disutility of depositors from volatility of the consumption and thus from variance of shocks. Therefore, depositors increase the share of savings in their income while decreasing the share of consumption. In deposit markets, higher supply of deposits induces lower interest rates. Thus banks increase the leverage as shown in Figure 1.7. Higher capital good supply implies a rise in output and wages which raise the

depositors' consumption. Low asset prices deteriorate banks' profits but since gains from lower borrowing rates overweight losses from asset price declines, shareholders' utilities increase with risk aversion of depositors.

#### ***1.5.3.3 Discount Factor ( $\beta$ )***

Figure 1.8 displays the effects of discount factor on the economy. An increase in discount factor implies higher valuation of future consumption by agents. Depositors save more and this raises the deposit supply in the markets. High deposit supply and thus low interest rates allow banks to increase their leverage. Higher lending stimulates the economy and output increases. Although depositors' saving ratio is higher, depositors' consumption increases as their wage income rises. On the other hand, excess supply of capital goods pulls asset prices down and deteriorates the profitability of banks. Moreover, increase in discount factor forces shareholders to invest more on bank capital and thus their consumption drops.

#### ***1.5.3.4 Variance and Persistency of Shocks***

An increase in the variance of shocks increases the savings of risk averse depositors. Such excess savings cause higher leverage and output as shown in Figure 1.9 and discussed in previous sections. In addition, higher variance of shocks increases the default risks and limits capital accumulation of the banks. As a result, banks' profits drop and shareholders' consumption decreases. Higher output implies higher wage income for the depositor. However excess volatility in wages and deposit incomes makes risk averse depositors save more by cutting their consumption. Hence, risk averse depositors' utilities decrease with higher variance of shocks.

An increase in persistency of shocks implies an increase in their variance, since shocks are defined as AR(1) process in equation (1.25). Therefore, the response of economy to an increase in persistency is similar to response to an increase in variance.

#### **1.5.4 Social Planner**

In this part of the quantitative analysis section, I evaluate simulation results of the social planner's problems and compare them with the competitive equilibrium.

##### ***1.5.4.1 Planner with Full Capability***

Planner's problem with full capability, given in equation (1.22), is solved using baseline calibration. Table 1.1 displays the unconditional moments of the lifetime utilities of agents. Compared with decentralized equilibrium, social planner almost doubles the loan size of an average bank and raises the output by 25 percent. It is also capable of increasing depositors' utilities while preserving the shareholder's utilities. Social planner can succeed such an improvement in the economy as it is able to make ex-post transfers to default banks and prevent idiosyncratic risk in the economy.

##### ***1.5.4.2 Planner with Restricted Capability***

As discussed in previous sections, planner with full capabilities is not a realistic reference point for possible government regulations. Therefore, planner with restricted capability is introduced in (1.23). There exist two types of solution to planner's problem; interior and binding, depending on the variance of the shocks in the model. Simulation results in Figure 1.10 displays

how depositors' utilities change with variance of shocks in restricted planner's problem and decentralized problem.

As it is shown in proposition 1, there exist a threshold level of variance where depositor's utility in planner's problem and decentralized equilibrium is the same. Simulations indicate that threshold level of standard deviation of shocks ( $\tilde{\sigma}$ ) is equal to 0.59 under baseline calibration. When the variance is higher than the threshold level, depositor's utility in planner's problem is higher than its competitive equilibrium level and thus planner's solution becomes an interior solution.

Figure 1.11 shows how the utilities of agents change with loan levels when the planner's solution is an interior solution. Restricted planner limits the leverage of banks compared to decentralized equilibrium. As the default probability drops, banks are able to accumulate more capital in the long run. In the stochastic economy, banks can reach the lending levels of competitive equilibrium but with a lower leverage level. Lower leverage level decreases banks' borrowing cost and improves the profitability. Thus shareholder's utility is higher than the competitive equilibrium level. Limiting the bank's leverage implies a drop in depositor's utility when there is no aggregate shock, at the steady state. But in stochastic environment, risk averse depositors like the robust structure of the banks. Low default rate of banks implies low exposure of depositors to bankruptcy costs. Therefore depositors ask lower interest rates from the banks. This stimulates the lending and thus output in the economy. If the variance of the shocks is large enough as in Figure 1.11, depositors' utilities in planner's solution will be higher than the competitive equilibrium level and decentralized equilibrium will be constrained inefficient.

Proposition 2 shows that constrained inefficiency of competitive equilibrium is independent of variance of shocks. Figure 1.12 displays the simulation results when the variance of shocks is lower than the threshold level and thus planner's solution is binding. Planner prefers cutting the leverage of banks but decline in leverage is limited by the utility of the depositors. Even with a limited decline of steady state lending, shareholders' profit still improves. The mechanism is similar to the interior solution case. But since the uncertainty in the economy is smaller, benefits of having more robust banks are lower and it limits further decline in the leverage. However planner's leverage choice is adequate to make banks' shareholders better off and keep depositor indifferent. Thus competitive equilibrium is constrained inefficient even with low variance of shocks.

## 1.6 Conclusion

This paper has developed a dynamic stochastic general equilibrium (DSGE) model to investigate the effects of asset prices on banks' leverage choices and default risks. Different from the existing literature of pecuniary externalities in financial markets, model includes endogenous borrowing limits, the existence of bank defaults and related bankruptcy costs. Banks borrow from depositor with standard debt contract using their assets as partial collateral. This implicitly limits banks' leverage and allows bank defaults in case of sudden drops in banks' revenues. Banks lend to capital good firms with perfect state contingent contract. Capital good firms sell their capital goods to consumption good firms in competitive market and transfer their revenues

to the banks. The role of consumption good firms is to create demand for capital goods and provide wage income to depositors and banks' shareholders by using their labor force in production. In the model, there is one to one match between capital good firms and the banks. Therefore heterogeneity of capital good firms implies heterogeneity of banks in loan revenues. Sudden drop in the capital good prices pulls down the banks' revenues and causes defaults among the banking sector. In case of default, depositors get all revenues of the banks after paying bankruptcy costs. Such bankruptcy costs amplify the inefficiencies due to pecuniary externalities.

The other key feature of the model is to analyze utilities of both depositors and shareholders in a closed economy framework that gives a complete picture of welfare analysis. In previous studies, depositors or financiers of the entrepreneurs are excluded from welfare analysis by open economy assumption. Therefore only bankers' utilities are included in the analysis.

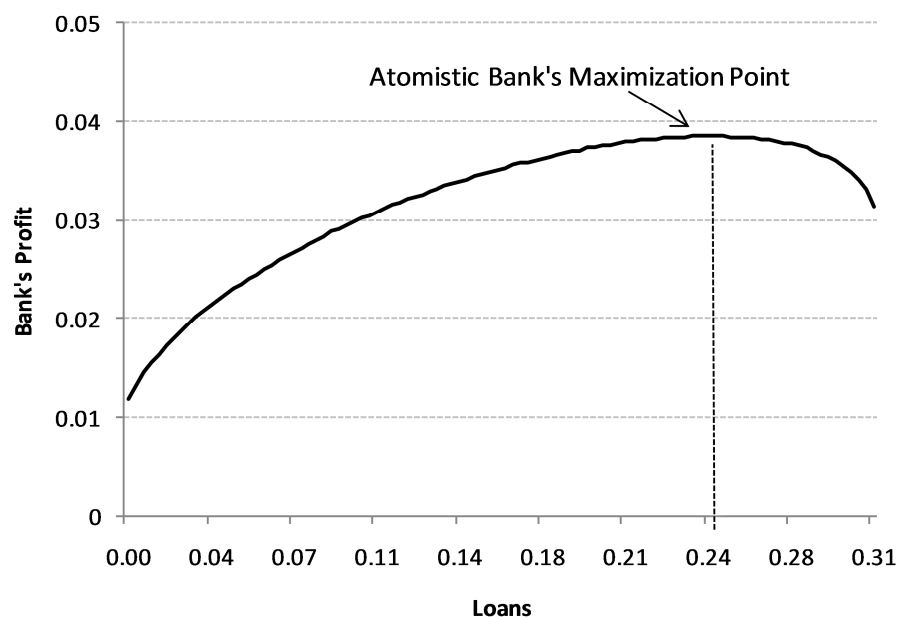
The key contribution of this paper is to show the existence of pecuniary price externalities and provide quantitative analysis of these externalities in an economy with aggregate and idiosyncratic uncertainty. I show that decentralized banks do not internalize effects of their lending decisions on distribution of asset prices and such pecuniary externalities leads banks to take excess risk by over-borrowing. Moreover, using both theoretical framework and quantitative analyses, I prove that even an extremely restricted social planner is able to make all agents better off by restricting the banks' leverage and risks in the economy. Our main conclusion is that generic inefficiencies of competitive equilibrium exist in financial markets regardless of the



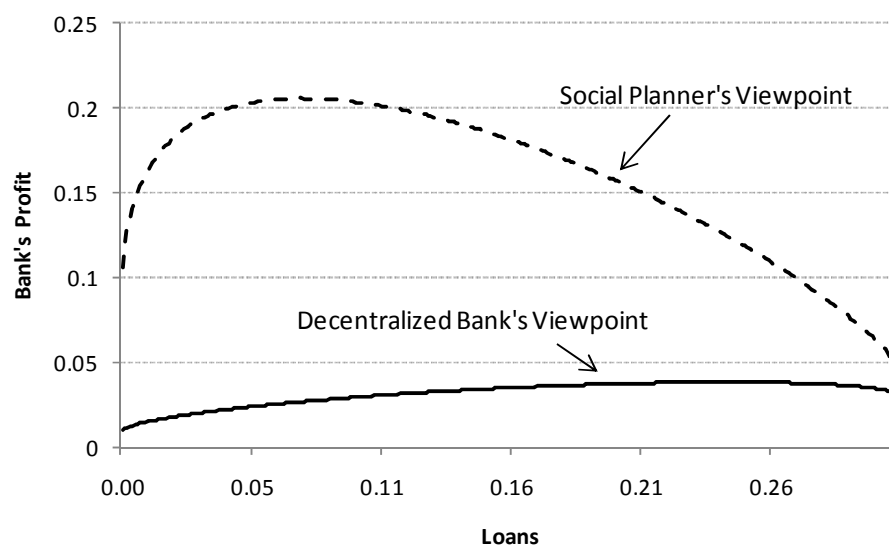
amplification mechanisms such as exogenous borrowing limits, and economy can be potentially better off with relevant regulations.

This paper also provides convenient framework for future studies to compare potential benefits of various policy implications to prevent pecuniary externality. One would like to examine the welfare results of different tax policies to limit excess leverage in financial markets. The model is also suitable to compare the effects of different bailout policies on agents' welfare. Another direction of future extension could be adding credit default swaps (CDS) mechanism into the model. Such an extension could wipe out the idiosyncratic shocks and limit the leverage if CDS are issued by banks. If they are issued by a third party that doesn't have adequate monitoring power on bank's activities, it might induce excess risk taking behavior among banks.

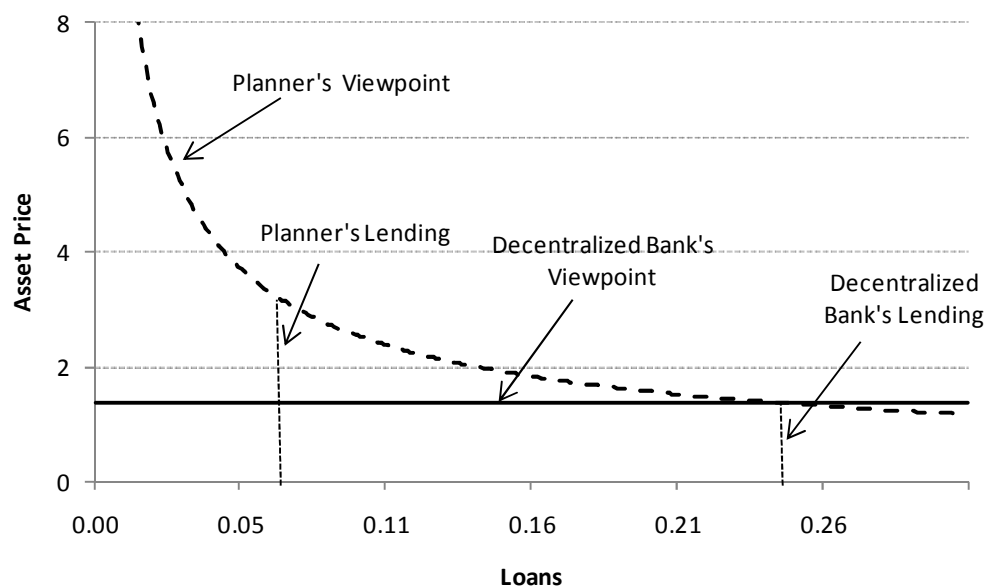
**Figure 1.1: Atomistic Bank's Profit Expectation in Financial Contract**



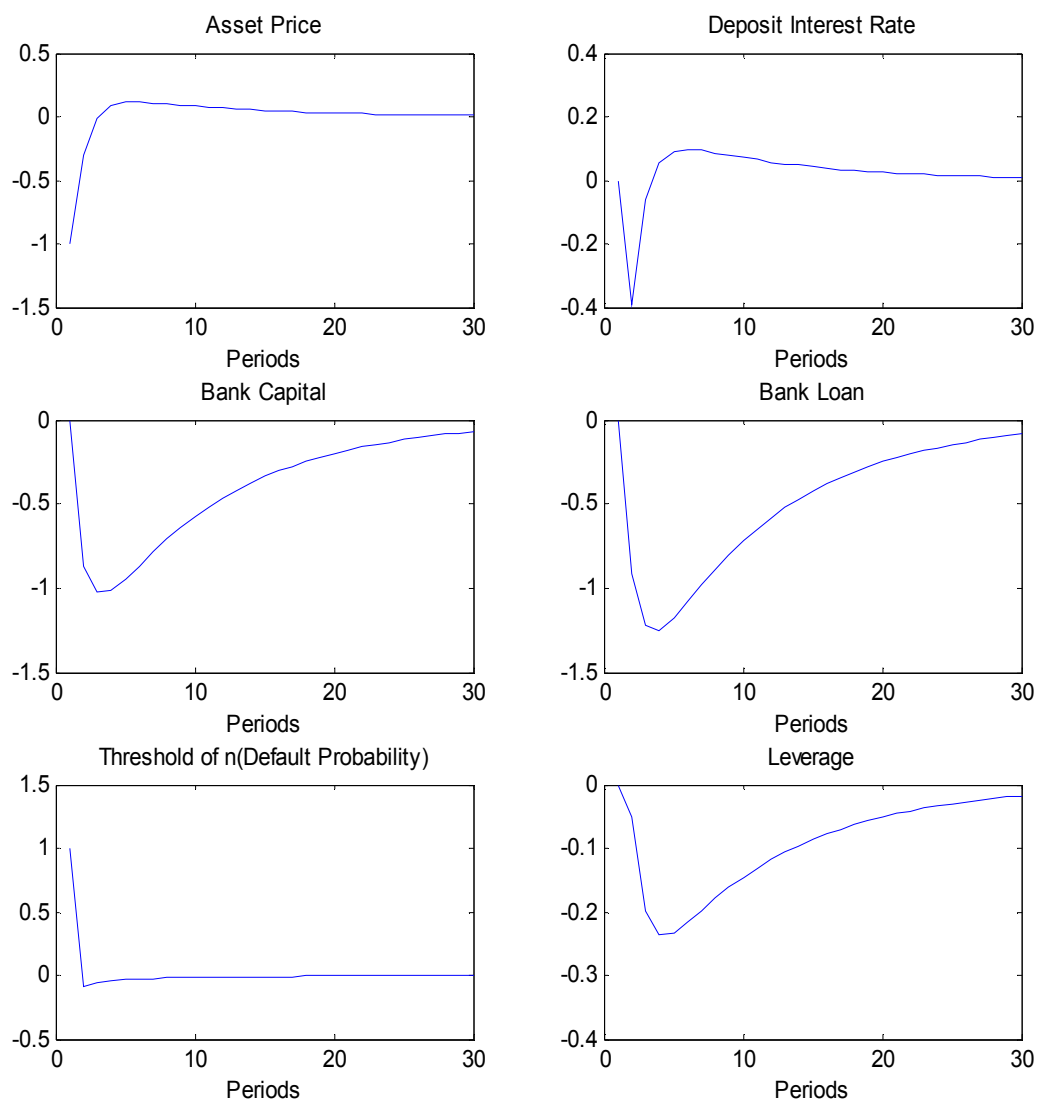
**Figure 1.2: Planner and Atomistic Bank's Perspectives for Bank's Profit Function  
(Exante)**



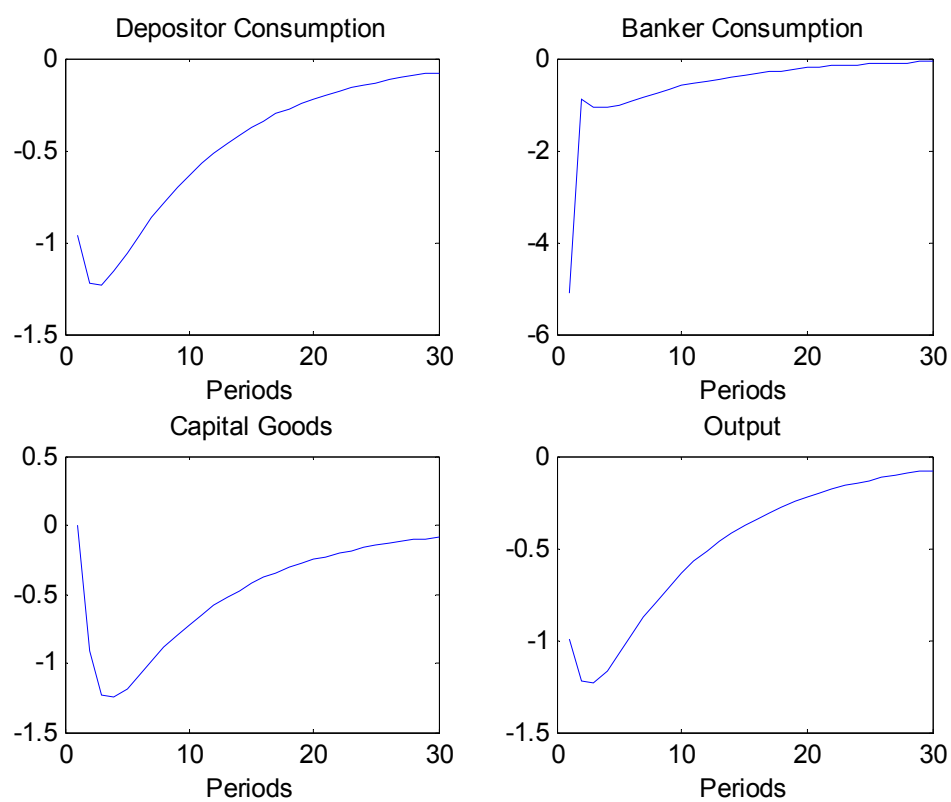
**Figure 1.3: Social Planner and Atomistic Bank's Perspectives for Asset Prices (Exante)**



**Figure 1.4: The Response of Financial Sector to Aggregate Productivity Shock**  
(percentage deviation from steady state)



**Figure 1.5: The Response of Economy to Aggregate Productivity Shock (percentage deviation from steady state)**



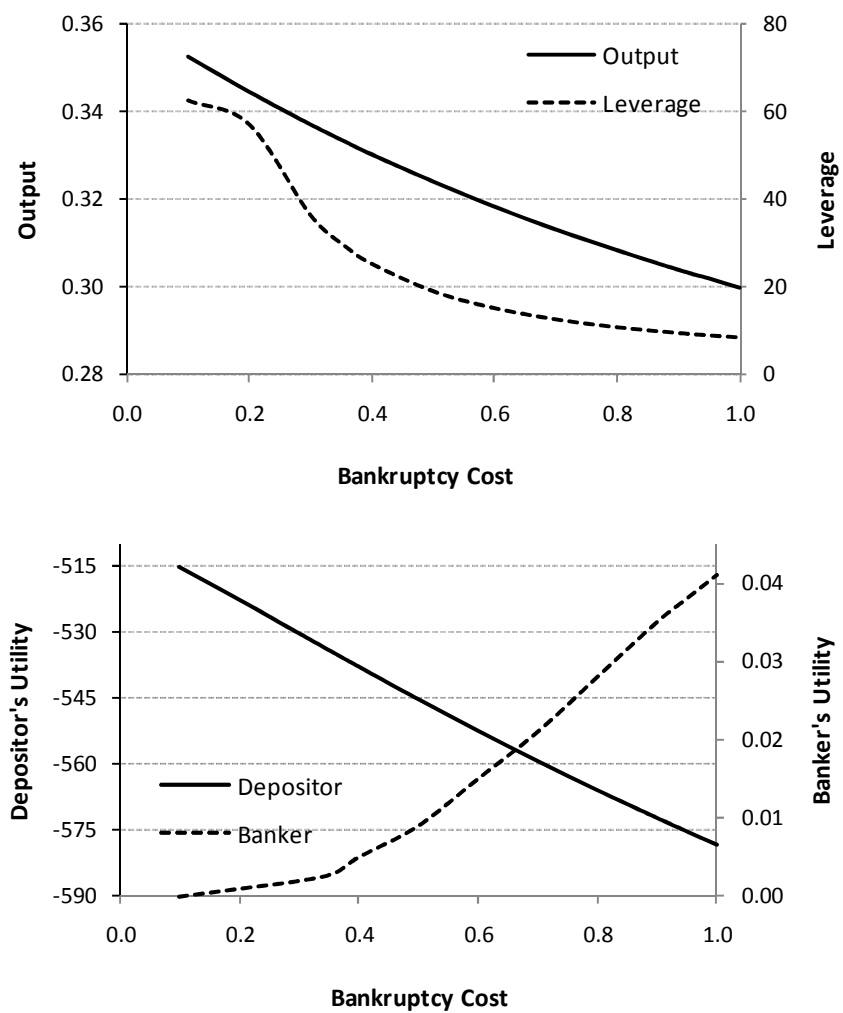
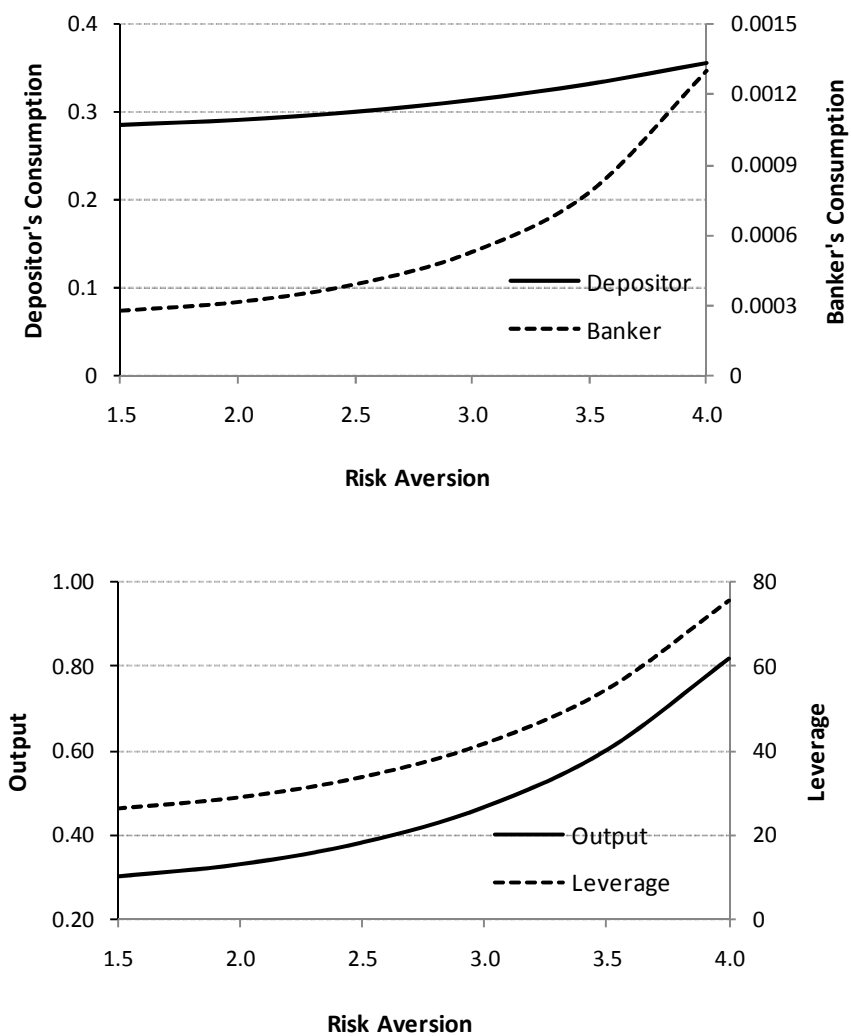
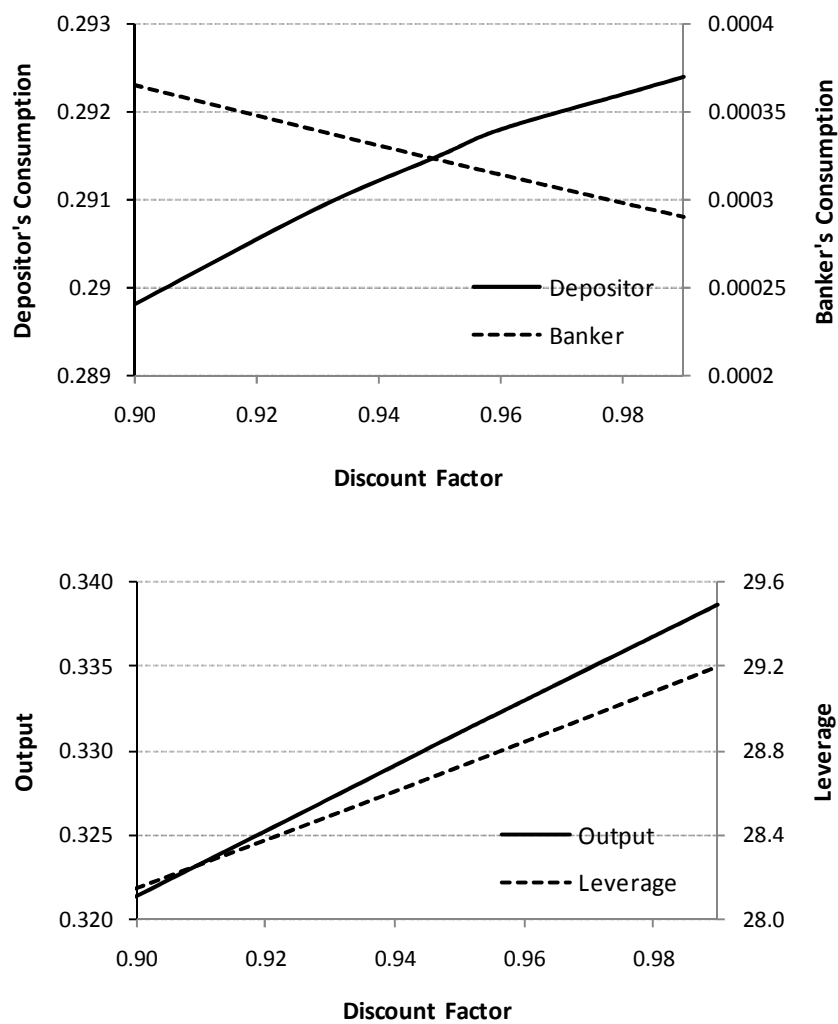
**Figure 1.6: Sensitivity of Economy to Bankruptcy Cost ( $\mu$ )**

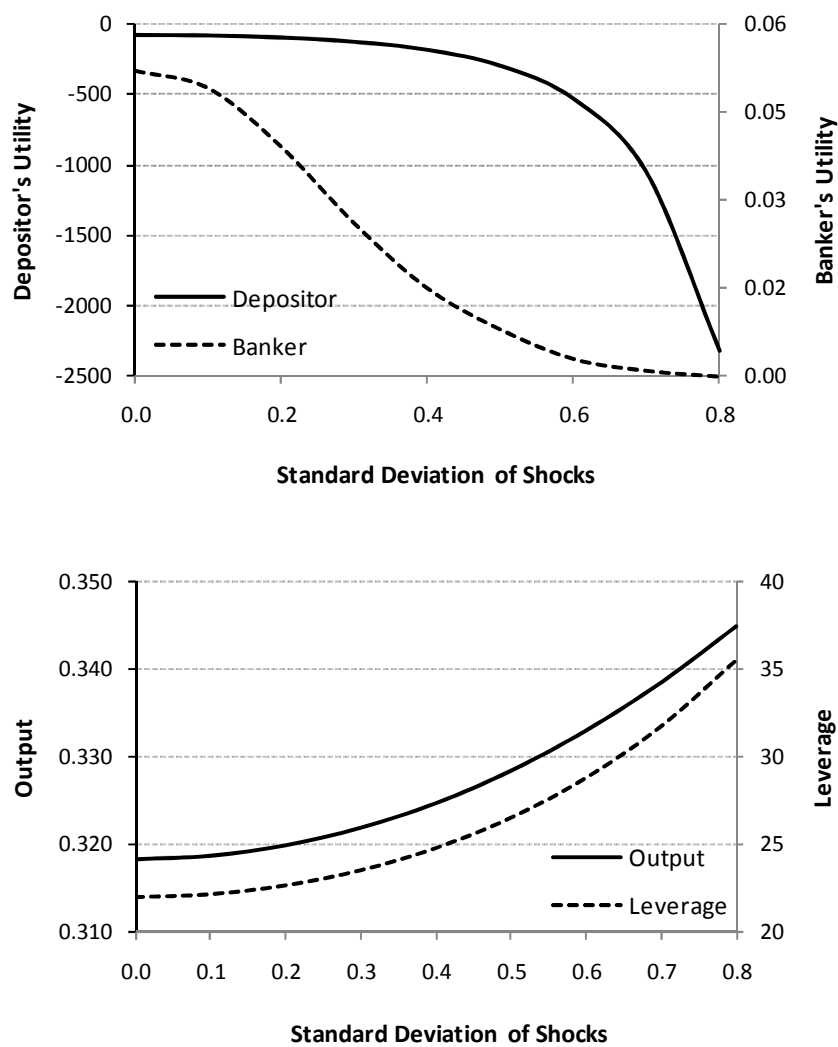
Figure 1.7: Sensitivity of Economy to Risk Aversion Parameter ( $\sigma$ )



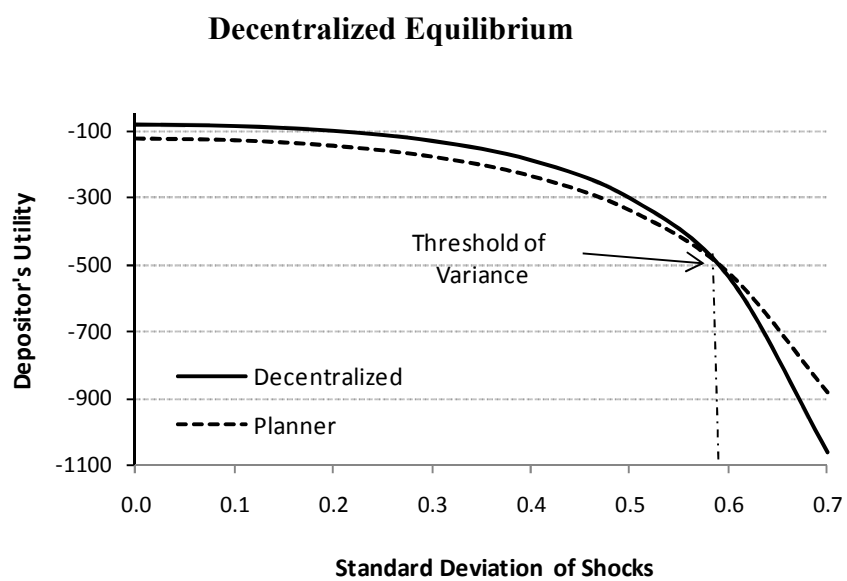
**Figure 1.8: Sensitivity of Economy to Discount Factor ( $\beta$ )**



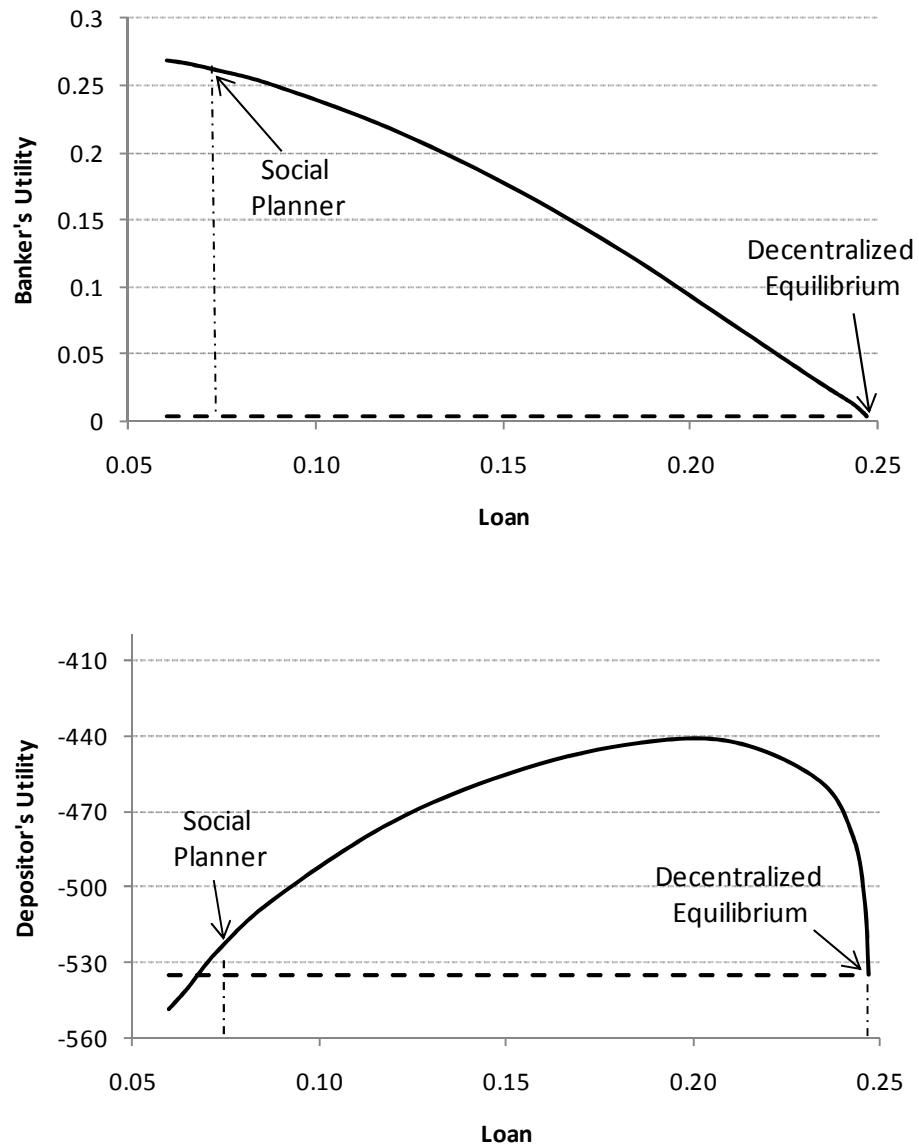


**Figure 1.9: Sensitivity of Economy to Variance of Shocks**

**Figure 1.10: Effects of Variance of Shocks on Depositor's Utility: Planner's Problem vs**

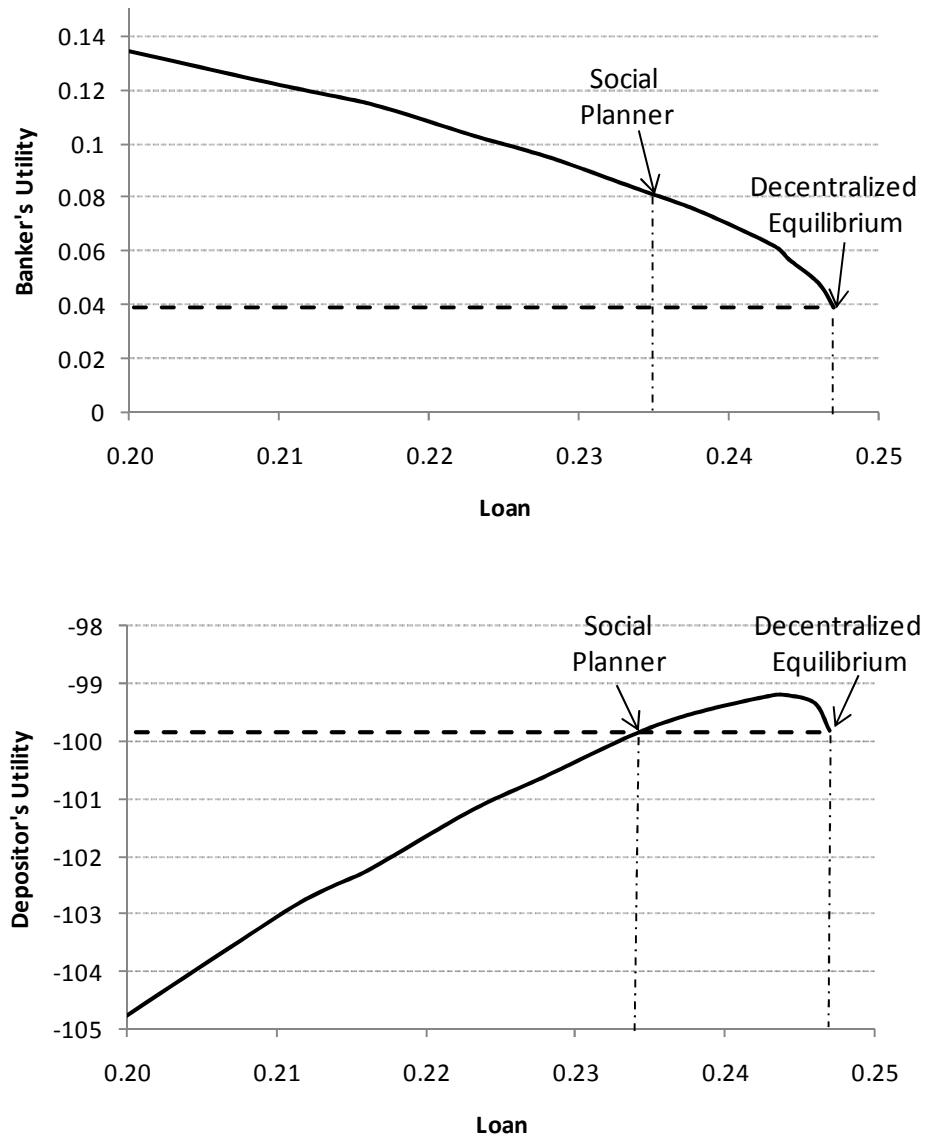


**Figure 1.11: Lifetime Utilities of Shareholders and Depositors ( $\sigma=0.6$ , Planner with Interior Solution)<sup>15</sup>**



<sup>15</sup> Loans are steady state levels and utilities are unconditional moments.

**Figure 1.12: Lifetime Utilities of Shareholders and Depositors ( $\sigma=0.2$ , Planner with Corner Solution)<sup>16</sup>**



<sup>16</sup> Loans are steady state levels and utilities are unconditional moments.

**Table 1.1: Social Planner with Full Capability vs Decentralized Equilibrium**

	Depositor's Utility	Banker's Utility	Loan	Output
Decentralized	-534.8	0.003	0.281	0.333
Social Planner	-442.8	0.003	0.526	0.415

## **Chapter 2**

# **The Welfare Analysis of Asset Price Externalities in Banking Industry**

### **2.1 Introduction**

The financial system including commercial banks, investment banks, hedge funds, etc. is the heart of economy. Their role is directing capital to where it is needed and where it is most productive in an effort to decrease inefficiency and risk in economy. The financial crisis in 2008, exacerbated by the excessive risk-taking and overborrowing of banks and their subsequent bailouts was a serious heart attack in both US and global economy. This heart attack was not an unexpected one; it was the result of unhealthy diet over many years. US governments counting

on the power of decentralized economy and free markets started to remove regulation on US economy. Regulations such as Glass-Steagall Act of 1933 are removed gradually starting at the end of 90s and they are replaced by self-regulation system which encouraged risk-taking. Moreover governments gave implicit and explicit bailout guarantees using various tools such as asset purchases and Federal Reserve discounting windows. However financial crisis in 2008 and following recession made it clear that self-regulation system in the financial sector doesn't work and banks ignore systemic risks especially in asset markets. Therefore, to prevent possible future financial crises, it is essential to re-examine the structure of the banking system and its relationships within the whole economy by comparing welfare analysis of different policies.

This paper presents welfare analysis of dynamic and stochastic economy including the banks with the agency costs and defaults at equilibrium. The banks are at the center of the dynamic stochastic general equilibrium (DSGE) model and unlike other models in the literature they can default with endogenous borrowing limits at equilibrium. We show that generic inefficiencies of competitive markets and agency problems in banks' management lead privately optimal decisions of bankers to high risk levels and over-borrowing at social level. We analyze society's welfare under various regulatory policies and find that policies which restrict lending ex-ante can restore constrained efficiency. Specifically, our results provide a justification for macro prudential approach in regulations. Unlike the existing literature, our paper provides quantitative assessments of welfare for both depositors and banks' shareholders in an economy with idiosyncratic and systemic risks.

The two key sources of inefficiency in our model are the agency problem and pecuniary price externalities. Agency problem exists since shareholders and banks' managements are different in the model. Shareholders provide capital to the banks and optimize their utilities in the long run. On the other hand banks' managements give lending and borrowing decisions and their goal is to optimize banks' profits in short run. Therefore bank's management undervalues the possible risks and damages from defaults. One of the contributions of our paper is to compare all welfare analysis and planner's solution from the second best perspective by keeping agency problem in the model. Second source of inefficiency is the pecuniary price externalities due to generic inefficiency of the competitive equilibrium. Private atomistic bankers do not internalize the effect of their lending decisions on asset prices and impose pecuniary externality on each other.

Our paper develops an infinite period model with incomplete financial markets. The key features of the model are endogenous borrowing limit, existence of default and related bankruptcy costs. Banks can borrow from depositors with standard debt contract using their initial capital as partial collateral. Unlike the existing literature of pecuniary externalities, instead of financial frictions and exogenous borrowing limits, we use partial collateralization mechanism which imposes endogenous borrowing limits and allows default of banks in the equilibrium.

In the model, banks are exposed to idiosyncratic risk from capital good firm's technology and systemic risk from the capital good prices. Banks are fully rational and correctly perceive the risk associated with their lending decisions. Nevertheless atomistic banks do not internalize their contribution to asset prices and impose pecuniary externality on each other. This prevents



atomistic banks to use the right asset price distribution in their ex-ante decisions, and induces them to take excessive systemic risk.

In this paper, we examine variety of regulations and bailout policies. These measures are imposed before a crisis hit so that we can assess the efficiency of policies against uncertainties in the economy. We find that policies that restrict lending in decentralized economy improve welfare of both depositors and banks' shareholders. Moreover social planner solution can't be restored by constant capital requirement as proposed in Basel requirements. We show that governments can reach constrained efficiency by implying reserve requirements. If the aggregate uncertainty is below the certain level, prudential reserve requirements will serve better than constant reserve requirements which justifies macro prudential policies in emerging countries such as Brazil and China. We study different bailout policies such as financed by tax on wages and tax on banks. We find that preannounced bailout policies exaggerate the risk appetite of banks and damage welfare of the economy.

Our model builds on the literature of macroeconomics models with financial frictions described by Bernanke and Gertler (1989), Bernanke, Gertler and Gilchrist (1999) and Carlstrom and Fuerst (1997). These papers and following work in the literature introduced agency costs into dynamic macroeconomic models to amplify the macroeconomic shocks. This paper is focused on asset pricing and banking sector balance sheet relations by eliminating the monetary authority and all other nominal frictions (price frictions). Different than the literature, first in this paper, effects of bankruptcy costs in a stochastic economy are considered by using second order approximation which is crucial in welfare comparisons.

Our paper is also related to the literature on generic inefficiency of competitive equilibrium under incomplete markets. (Hart, 1975; Stiglitz, 1982; Geanakoplos and Polemarchakis, 1986). The more recent contributions in this literature (Lorenzoni, 2008; Korinek, 2011; Jeanne and Korinek, 2011; Bianchi, 2010) analyzed the role of pecuniary externalities in the inefficiencies of decentralized equilibrium under financial frictions. Lorenzoni (2008) and Korinek (2011) used finite period model in which entrepreneurs borrow to finance their projects in the first period and face the risk of binding financial constraints in the following period. Entrepreneurs' defaults are excluded by allowing the fire sales of productive assets. In financially binding states fire sales cause pecuniary externality in asset prices, which is ex-ante undervalued by atomistic entrepreneurs. Pecuniary externalities exist only in fire sales of productive assets because entrepreneurs are not allowed to sell their production in the markets. In this paper, by contrast, all production goods can be sold in the markets, and furthermore, banks will default if they can't pay their debt. Thus, we are able to analyze the effects of pecuniary externalities in all states, and moreover, we show that generic inefficiencies of decentralized equilibrium still exist without amplification effects of fire sales in binding states.

Bianchi (2010) and Jeanne and Korinek (2011) analyzed similar pecuniary externalities using open economy DSGE models. In these papers, insiders borrow from outsiders and their default is excluded by full collateralization. Borrowing limits depend on asset prices with exogenous parameters. Borrowing limits are binding only in bad states and they amplify the effects of crisis by depressing the asset prices more. Atomistic agents overborrow by ignoring pecuniary externalities and this is the source of inefficiency in the economy. In these studies,

default is not allowed and pecuniary externality exists only in financially constrained states. Furthermore, outsiders' utilities are excluded by open economy assumption in welfare analysis. In this paper, by contrast, there is partial collateralization in a closed economy and borrowing limits are endogenously determined in the market. Thus, I can analyze bank defaults and consider the welfare of all agents in the model. Moreover, in this study, pecuniary externalities exist in all states of the economy.

The rest of the paper is organized as follows. Section 2 introduces the benchmark model. Section 3 characterizes the competitive equilibrium. Section 4 analyzes the social planner's problem. Section 5 presents the quantitative analysis. Section 6 compares the social welfare under different policy measures and presents sensitivity analysis and section 7 concludes.

## **2.2 The Model**

The model is a closed economy DSGE model and it consists of continuum of agents with unit mass. Basic structure of the model as follows: There are two types of atomistic agents; depositors (fraction  $1-\eta$ ) and banks' shareholders (fraction  $\eta$ ). Depositors are the identical households living forever. They work, save and consume. Shareholders are the owners of the banks and each shareholder has shares in a specific bank. Shareholders have two sources of income; labor income and profit of the banks. They can either consume or save by investing in

banks as capital. Both shareholders and banks are heterogeneous in terms of their initial capital.<sup>1</sup>. Banks' managements are different than shareholders. They have short term, one period, contract and their objective, given the initial capital, is to maximize banks' profits at the end of each period. At the end of each period, banks borrow from depositors to invest in next period's capital good production. While borrowing, banks' managements solve the financial contract problem. Since financial contract is just one period in length, it is explained separately from general equilibrium model.

Banks have two sources for their lending; their deposits ( $d_t$ ) and capital ( $w_t$ ). Atomistic depositors are the only suppliers of deposits. Depending on the average expected return ( $a_t$ ) from each unit of deposit, depositor saves  $d_t^s$  unit of consumption good and distributes his savings among the banks. Law of large numbers allows them to insure against idiosyncratic risk of the banks' lending activities. But depositors are still exposed to the banks' default risk due to systemic risk. Banks pay interest rate  $r^d$  when they do not default. When bank defaults, depositors get all the remaining capital of the bank and pay the bankruptcy cost, defined as monitoring and liquidation costs. In the deposit markets,  $r_t^d$  and  $a_t$  clear the market and equalize total saving of depositors to total deposit demand from the banks,  $(1-\eta)d_t^s = \eta d_t$ .<sup>2</sup>

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<sup>1</sup>Keeping track of variables' mean instead of their distribution is adequate due to the assumption of atomistic agents and law of large numbers.

<sup>2</sup> Deposit demand is different among the heterogeneous banks. But assumption of law of large numbers allows us using  $d_t$  (deposit demand of average bank) instead.

Banks' revenues depend on the performance of their investment in capital good firms. Capital good firms' initial endowment is zero. In order to produce, they need to borrow  $l_t$  units of loan. There is one to one match between banks and firms. Each firm borrows only from a specific bank. Their technology is linear,  $n_t l_t$ .  $n_t$  is the productivity factor and randomly distributed over the interval  $[n^L, n^H]$  with the density function  $f(n_t)$ , assumed uniform. Both banks and capital good firms don't know productivity level before production. The banks have perfect monitoring skills over firms. The contract between firms and the banks is assumed perfectly state contingent debt contract similar to issuing equity for debt. Capital good firms sell capital goods to consumption good firms at price  $q_t$  and transfer all their returns to the banks.

If bank's revenues are not enough to pay its liabilities to depositors ( $q_t n_t l_t < r^d d_t$ ), it will default. Productivity parameter of capital good firms is crucial to determine default risk. There exists a threshold level of  $n$  such that any realization of productivity below this threshold causes default.

$$\tilde{n}_t = \frac{r_t^d d_t}{q_t (d_t + w_t)} \quad (2.1)$$

Banks' default probability is the cumulative distribution function of  $n$  at the threshold level,  $F(\tilde{n}_t)$ . Under the financial contract with given  $q_{t+1}$ , banks' expected profit (in terms of consumption goods) for  $t+1$  at period  $t$  is:

$$q_{t+1} l_{t+1} \pi_{t+1}^B = q_{t+1} l_{t+1} \left( \int_{\tilde{n}_{t+1}}^{n_H} n_{t+1} f(n) dn - (1 - F(\tilde{n}_{t+1})) \tilde{n}_{t+1} \right) \quad (2.2)$$

where  $\pi_{t+1}^B$  is the bank's return from each unit of lending in terms of capital good. Similarly a bank's expected total payment to all its depositors, given  $q_{t+1}$  is:

$$q_{t+1} l_{t+1} \pi_{t+1}^D = q_{t+1} l_{t+1} \left( \int_{n_L}^{\tilde{n}_{t+1}} (1-\mu) n_{t+1} f(n) dn + (1-F(\tilde{n}_{t+1})) \tilde{n}_{t+1} \right) \quad (2.3)$$

where  $\mu$  is the bankruptcy cost parameter and  $\pi_{t+1}^D$  is the depositors' return from each unit of lending in terms of capital good. At the end of each period, banks optimize next period's expected profits subject to depositors' participation constraint. . The optimal contract is the solution of the bank's optimization problem given by:

$$\max_{l_{t+1}} E_t q_{t+1} l_{t+1} \pi_{t+1}^B, \text{ subject to } E_t q_{t+1} l_{t+1} \pi_{t+1}^D \geq a_{t+1} d_{t+1} \quad (2.4)$$

Depositors are identical and their participation constraint is always binding. It is assumed that returns are attractive enough to convince banks to borrow. Therefore, banks' participation constraint is always satisfied and can be ignored while solving the optimization problem. It is important to mention that shareholders own the banks but it is the bank's management who is in charge and solves the contract problem. Since managements' employment contract is short term, their goal is to maximize the next period profits and they ignore further future profits (agency problem). This induces risk taking behavior of banks as future losses in case of possible defaults are ignored in the financial contract.

In the general equilibrium model, infinitely-lived, identical, atomistic depositors' preference is given by:

$$U^D = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^D) \quad (2.5)$$

In this expression,  $\beta$  is the discount factor and utility of depositor has the constant-relative-risk-aversion (CRRA) form. Each period, depositors allocate their incomes between saving and consumption according to budget constraint given by:

$$d_{t+1}^s + c_t^D \leq w_t^d h_t + q_t l_t \pi_t^D \eta / (1 - \eta) \quad (2.6)$$

where  $c_t^D$  is the consumption,  $h_t$  is the labor supply and  $w_t^d$  is the real wage. Labor supply is assumed constant to focus on financial relations. The second term at the right side of the budget equation is the return from savings. Since depositors are identical it is calculated by averaging the total payments from the banks.

Banks' shareholders have two sources of income; banks' profits and wage income. At the end of each period, they decide how much of the income to consume and reinvest in banks. Their reinvestments are used as initial capital for banks' next period operations. In order to induce shareholders' consumption and thus banks' borrowing in the model, we assumed that shareholders discount future more heavily than households. Shareholders' preference is given by:

$$U^B = E_0 \sum_{t=0}^{\infty} (\beta \gamma)^t c_t^B \quad (2.7)$$

where  $c_t^B$  denotes shareholder's time  $t$  consumption and  $\gamma \in (0,1)$  denotes the additional discount rate. Shareholders supply constant labor each period and this allows them to enter the banking system next period even if they default. Shareholders' budget constraint is given by:

$$w_{t+1} + c_t^B \leq q_t l_t \pi_t^B + w_t^B \quad (2.8)$$

where  $w_{t+1}$  is the next period bank's capital and  $w_t^B$  is the shareholder's wage.

Consumption goods producers buy capital from capital goods firm and hire labor from depositors and shareholders. Capital is fully depreciated at the end of each period. Their production technology is given by:

$$Y_t = \theta_t (K_t)^\alpha (H_t)^{1-\alpha_H} (\eta)^{\alpha_H} \quad (2.9)$$

where  $\theta_t$  denotes the stochastic productivity parameter,  $K_t$  denotes the aggregate supply of capital goods,  $H_t$  denotes the aggregate supply of depositors' labor and  $\eta$  denotes the aggregate supply of shareholders' labor.

## 2.3 Equilibrium

### 2.3.1 Optimality Conditions

In the financial contract (2.4), bank's problem at time  $t$  is to choose  $l_{t+1}$  to maximize its expected profit subject to depositor's participation constraint. The optimality conditions require<sup>3</sup>:

$$q_{t+1} (\bar{n} - \mu \frac{(\tilde{n}_{t+1})^2 - (n^L)^2}{2(n^H - n^L)} + \mu \frac{\tilde{n}_{t+1}}{n^H - n^L} \frac{\pi_{t+1}^B}{\pi_{t+1}}) = a_{t+1} \quad (2.10)$$

$$q_{t+1} l_{t+1} \pi_{t+1}^D = a_{t+1} d_{t+1} \quad (2.11)$$

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<sup>3</sup> Detailed solution of financial contract is given in Appendix A.1.



where  $\bar{n}$  is the expected return. Optimality condition (2.10) equates marginal profit of bank from an extra unit of lending to the marginal cost of borrowing. As the threshold level of  $n$  depends on interest rate and leverage ratio in equation (2.1), optimality conditions (2.10 and 2.11) imply that all banks choose same leverage ratio in the financial contract.<sup>4</sup> It allows great simplification in the solution of the general equilibrium model while preserving the heterogeneity among the banks.

In the general equilibrium model, depositors choose the stochastic processes  $\{c_t^D, d_{t+1}^s\}_{t \geq 0}$  to maximize their utility (2.5) subject to their budget constraint (2.6). The optimality conditions require<sup>5</sup>:

$$u_{c,t} = \beta E_t u_{c,t+1} \left( (1-F(\tilde{n}_{t+1}))r_{t+1}^d + (1-\mu)q_{t+1} \frac{l_{t+1}}{d_{t+1}} \int_{n_L}^{\tilde{n}_{t+1}} n_{t+1} f(n) dn \right) \quad (2.12)$$

$$d_{t+1}^s + c_t^D = w_t^d h_t + q_t l_t \pi_t^D \eta / (1-\eta) \quad (2.13)$$

The optimality condition (2.12) is the euler equation. Expression in the brackets is the depositor's next period's return from extra unit of saving.

Shareholders choose the stochastic processes  $\{c_t^B, w_{t+1}\}_{t \geq 0}$  to maximize their utility (2.7) subject to the budget constraint (2.8). The optimality conditions are given by:

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<sup>4</sup> Detailed proof is given in Appendix A.1.

<sup>5</sup> Detailed derivation of depositors and bankers optimality conditions are given in Appendix A.

$$\frac{1}{\beta\gamma} = q_{t+1} \pi_{t+1}^B \frac{l_{t+1}}{w_{t+1}} \quad (2.14)$$

$$w_{t+1} + c_t^B = q_t l_t \pi_t^B + w_t^B \quad (2.15)$$

Equation (2.14) is the euler equation for the shareholders. Expression at the right side of equation is the bank's return from extra unit of capital.

There are four markets; deposit, labor, capital goods and consumption goods. Competition in the factor markets implies that wages and capital good prices are equal to their marginal products:

$$w_t^d = (1 - \alpha - \alpha_H) \frac{Y_t}{H_t} \quad \text{and} \quad w_t^B = \alpha_H \frac{Y_t}{\eta_t} \quad (2.16)$$

$$q_t = \alpha \theta_t (K_t)^{\alpha-1} (H_t)^{1-\alpha-\alpha_H} (\eta_t)^{\alpha_H} \quad (2.17)$$

Technology parameter of the consumption good firms,  $\theta_t$ , is the source of systemic risk in the economy via capital good prices. Labor, deposit, capital goods and consumption goods market clearing conditions are given by<sup>6</sup>:

$$H_t = (1 - \eta) h_t \quad (2.18)$$

$$K_t = \eta \bar{n} l_t \quad (2.19)$$

$$(1 - \eta) d_t^s = \eta d_t \quad (2.20)$$

$$Y_t = (1 - \eta) c_t^D + \eta c_t^B + \eta F(\tilde{n}_t) \mu q_t l_t + \eta l_{t+1} \quad (2.21)$$

---

<sup>6</sup> Aggregate values are calculated by multiplying the fraction of agents in population with the average value of variables.

### 2.3.2 Equilibrium Definition

There are five state variables in the decentralized optimization problem. At period  $t$ , banks and depositors decide on next period's lending ( $l_{t+1}$ ) and starting capital ( $w_{t+1}$ ). In deposit markets, depositor's average expected future return ( $a_{t+1}$ ) is determined to clear the deposit market. Similarly deposit interest rate ( $r_{t+1}^d$ ) is determined to insure the depositors. The other state variable is the technology of consumption good firms. Rational agents perceive actual levels of aggregate lending and capital. They estimate asset price using equation (2.17).

A decentralized competitive equilibrium for this economy is defined by interest rate  $r_{t+1}^d$ , deposit return  $a_{t+1}$ , wages  $w_t^D$  and  $w_t^B$ , asset price  $q_t$  and decision rules for  $c_t^D$ ,  $c_t^B$ ,  $w_{t+1}$ ,  $l_{t+1}$ ,  $d_{t+1}$ ,  $K_{t+1}$ ,  $\tilde{n}_t$  where they are the functions of  $(r_t^d, a_t, l_t, w_t, \theta_t)$  and satisfy the following:

- (i) Financial contract optimality conditions (equation 2.10 and 2.11).
- (ii) Depositors' optimality conditions (equation 2.12 and 2.13).
- (iii) Bankers' optimality conditions (equation 2.14 and 2.15).
- (v) Market clearing conditions (equation 2.16 to 2.21).

Equilibrium of such decentralized model exists and it is unique. Detailed proof is given in Appendix B.

## 2.4 Social Planner's Problem

In the decentralized equilibrium, atomistic agents take aggregate variables as given, particularly total lending ( $L_{t+1}$ ). While solving the financial contract, individual banks don't internalize their contribution to total lending ( $dL_{t+1}/dl_{t+1}=0$ ) which implies that banks ignore the effects of their actions on the capital good's price distribution ( $dq_{t+1}/dl_{t+1}=0$ ). Thus atomistic banks overvalue the marginal gains from lending in their optimality condition (2.10). This induces atomistic banks to take excess risk by lending more, and hence high default rates are observed at the decentralized equilibrium.

### 2.4.1 Planner with Restricted Capability

Excess capability of planner is not a suitable reference point for possible government regulations. Therefore, in the remainder of the paper, we assume planner's abilities are limited and we analyze solution from second best perspective. We suppose that planner has control only over the banking sector and it can't make ex-post transfers to the banks and depositors<sup>7</sup>. We assume that planner has regulatory tools to control the banks' lending in the financial contract. Thus planner can interfere only in financial contracts on behalf of the banking sector.

The social planner solves the financial contract problem (2.4) by internalizing effects of lending decisions on asset prices. The optimization problem is formulated as:

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<sup>7</sup> This assumption will be relaxed in the following sections while testing various policy recommendations.

$$\begin{aligned}
& \max_{l_{t+1}} E_t q_{t+1} l_{t+1} \pi_{t+1}^B \\
& \text{subject to} \\
& E_t q_{t+1} l_{t+1} \pi_{t+1}^D \geq a_{t+1} d_{t+1} \\
& E_t q_{t+1} = E_t \alpha \theta_{t+1} (\bar{n} l_{t+1})^{\alpha-1} (H_{t+1})^{1-\alpha-\alpha_H} (\eta)^{\alpha_H} \\
& U^D \geq (U^D)^{DE}
\end{aligned} \tag{2.22}$$

where  $(U^D)^{DE}$  represents the lifetime utility level of average depositor in decentralized equilibrium. In order to have an interior solution, we assume that depositors' utility constraint ( $U^D \geq (U^D)^{DE}$ ) is not binding.<sup>8</sup> Given  $\theta_{t+1}$ , optimality condition is given by:

$$\alpha q_{t+1}^S (\bar{n} - \mu \frac{(\tilde{n}_{t+1})^2 - (n^L)^2}{2(n^H - n^L)} + \mu \frac{\tilde{n}_{t+1}}{n^H - n^L} \frac{\pi_{t+1}^B}{\pi_{t+1}^D}) = a_{t+1} \tag{2.23}$$

where  $q_{t+1}^S$  is the capital good price in social planner's equilibrium. Comparing equation (2.10) and equation (2.23), planner perceives lower marginal benefit from excess lending levels of decentralized banks. Planner internalizes its contribution on asset price distribution while atomistic banks assume that prices are independent of their lending. Thus planner chooses lower lending than the decentralized banks. The decline in banks' lending implies a decline in supply of capital goods which causes higher asset prices in planner's problem ( $q_{t+1}^S \geq q_{t+1}^{DE}$ ). Therefore, in planner's problem, even banks' lending is lower, banks' profits are higher than the decentralized equilibrium and risk neutral shareholders are always better off.

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<sup>8</sup> In Appendix D and in quantitative analysis section, I show that restricted planner solution is still pareto better when depositors' utility constraint is binding and there is corner solution.

## 2.5 Quantitative Analysis

In this section, we describe the quantitative implications of the model. While solving the model numerically, we use second order approximation with perturbation method. Since the model is set up in stochastic environment, second order approximation improves the accuracy of the model and estimations (Schmitt-Grohe and Uribe, 2004).

The model is parameterized using standard values in the related literature. The risk aversion parameter,  $\sigma$ , is set at 2. Quarterly discount factor  $\beta$  is set at 0.96, implying 4 percent annual average return on deposits at steady state. In consumption good production, the capital share ( $\alpha$ ) is set at 0.35. Labor share of depositors and shareholders are set at 0.64 and 0.01 respectively. The shareholders' labor share ensures that shareholders have positive net worth even after default to make a fresh start. It also guarantees the positive consumption of shareholders in case of default.

Productivity factor of capital good firm,  $n$ , is randomly distributed over the interval  $[0,2]$  with the uniform density function as in Carlstrom and Fuerst (1997). The variance of consumption good firm's technology parameter ( $\theta$ ) is set to 0.36, in keeping with the literature. Results with different levels of variance are provided in sensitivity analysis. The ratio of shareholders in population doesn't have significant effect on the results and it is set at  $\eta = 0.33$ .

As for the bankruptcy cost parameter, there is considerable controversy in the literature. Among the similar class of models, Carlstrom and Fuerst (1997) defined the reasonable interval as  $(0.2, 0.36)$  by referring to the empirical studies of Altman (1984) and Alderson and Betker

(1995). Bernanke, Gertler and Gilchrist (1999) set the parameter to 0.12. All of these studies defined the bankruptcy costs for a classical firm rather than a financial institution. The bankruptcy cost is underestimated for the banking sector considering the Lehman case in 2008, the largest bankruptcy filed with \$ 691 billion in assets. According to FDIC report<sup>9</sup>, current bankruptcy plan can recover only 20 cents on every \$ 1 of claims and the optimistic completion date is 2014, 6 years after the bankruptcy filing. In the benchmark model, I set bankruptcy cost to 0.36, upper level in the literature. But in sensitivity analysis results with higher bankruptcy costs are provided.

The final parameter to be selected is additional discount rate of bankers ( $\delta$ ). This parameter allows shareholders to discount future more than depositors and induces the lending in the model. The focus of the paper is the leverage of the banks, so contrary to literature discount parameter is selected to match the leverage ratio of financial institutions. Leverage ratio used in Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999) is 2.5 and 0.5, respectively. Considering that average leverage of US banks varies between 20 and 40 over time (from 1992 to 2008)<sup>10</sup>, leverage ratios used in the literature are too low for financial institutions and they target classical firms instead. Since the focus of paper is financial institutions, discount rate is set to  $\delta=0.3$  which implies leverage at 22.

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<sup>9</sup> ‘The Orderly Liquidation of Lehman Brothers Holdings Inc. under the Dodd-Frank Act’, FDIC Quarterly (2011).

<sup>10</sup> Average leverage levels of US banks are reported since 1992 in Adrian and Shin (2010).

### 2.5.1 Planner with Restricted Capability

Simulation results in Figure 2.1 displays how agents' utilities change with variance of shocks in restricted planner's problem and decentralized problem. When the volatility in the economy is high enough, both depositors and banks' shareholders are pareto better in planner's problem. If the standard deviation of shocks is lower than the threshold level, 0.6, restricted planner can still make shareholders better off and keep depositors indifferent.

In stochastic environment, private banks couldn't estimate the costs of systemic shocks and take excess risk. But restricted planner internalizes these risks and limits the leverage of banks compared to decentralized equilibrium. As the default probability drops, banks are able to accumulate more capital in the long run. In the stochastic economy, banks can reach the lending levels of competitive equilibrium but with a lower leverage level. Low default rate of banks implies low exposure of depositor to bankruptcy cost. Therefore depositor asks lower rates from the banks and this stimulates the lending and thus output in the economy.

## 2.6 Welfare Analysis

In the previous chapter, we prove that decentralized equilibrium is not socially efficient and even a restricted planner can improve the welfare of all agents in the economy. Moreover we show that planner can succeed such a solution with lower leverage and default levels. In this chapter, we expand our analysis from restricted social planner to possible policy measures which are currently used by many countries and compare the efficiency of these measures.



### 2.6.1 Constant Reserve Requirement

Reserve requirement policies are especially popular in emerging countries to control the lending in banking system. In this section, we apply constant and prudential reserve requirements to the decentralized economy and compute the welfare gains.

Reserve requirement can be formulated like tax on deposits. In our model, we set it as tax on lending for simplification of calculations. Since initial capital is given, putting a tax on loans is similar to putting a tax on the deposits at the margin. In addition, we assume that after collecting reserve requirements, government uses them to subsidy banks with lump sum rebates. Such a policy limits the risk appetite of the banks as the cost of extra lending increases at the margin. Both depositors and banks benefit from such a policy. It provides less volatile deposit income for depositors as banks become more robust to systemic price shocks. Banks are better off too as higher asset prices increases their returns from investments. Moreover, low default rates decrease their funding costs.

The loan level under constant reserve requirement from banks' perspective is rewritten as;

$$l'_{t+1} = (1-\tau)l_{t+1} + T_{t+1} \quad (2.24)$$

where  $T_{t+1}$  is the lump-sum rebate,  $\tau$  is the constant reserve requirement and  $T_{t+1} = \tau l_{t+1}$ .

Private banks perceive cost of the loans higher due to reserve requirement. Banks' profit function in equation (2.2) is rewritten as;

$$q_{t+1} l'_{t+1} \pi_{t+1}^B = q_{t+1} l'_{t+1} \left( \int_{\tilde{n}_{t+1}}^{n_H} n_{t+1} f(n) dn - (1-F(\tilde{n}'_{t+1})) \tilde{n}'_{t+1} \right) \quad (2.25)$$

Solving financial contract in equation (2.4) using the new profit function with reserve requirement gives the first order condition as;

$$(1-\tau)q_{t+1}^S (\bar{n}-\mu \frac{(\tilde{n}_{t+1})^2-(n^L)^2}{2(n^H-n^L)} + \mu \frac{\tilde{n}_{t+1}}{n^H-n^L} \frac{\pi_{t+1}^B}{\pi_{t+1}^{B'}}) = a_{t+1} \quad (2.26)$$

If first order condition (2.26) is compared with social planner's first order condition in equation (2.23), constant reserve requirement can achieve restricted planner's solution and thus welfare levels when  $\tau = 1-\alpha$ . Notice that this reserve requirement level is optimal when the standard deviation of systemic shocks is above threshold level, 0.6 and hence solution is interior. As shown in planner's problem, when standard deviation is lower than the threshold level, problem has binding constraint. In this case, in order to find a solution, we apply a heuristic approach. First, using the simulations, we find the utility of shareholders and depositors for various levels of constant reserve requirement levels as displayed in Figure 2.2. Then we determine set of pareto better reserve requirement levels given as shaded region in Figure 2.2. Finally, within this pareto better range of reserve requirements, we keep depositors utility same as in decentralized economy and try to maximize shareholders' utility. Therefore we implicitly transfer all surpluses to shareholders. According to Figure 2.2, if we apply this approach, optimal reserve requirement level should be 10%. We use the similar approach in the following policy analyses.

### 2.6.2 Prudential Reserve Requirement

The other reserve requirement policy commonly preferred by many countries is macro prudential approach. In this policy, reserve requirement level is not constant; instead it is linear function of the state of economy. If the economy is heating, government will increase the reserve requirement to cool the economy and prevent excess lending of financial sector. If the economy is below its potential, government will cut reserve requirements to stimulate the lending. The mechanism is similar to constant reserve requirement policy; both depositors and banks benefit from more stable economy with lower default rates and higher asset prices. Moreover prudential policy will provide flexibility to the economy in stochastic environment against the persistency of shocks. Therefore both depositor and bankers welfare is better off.

In order to simulate the policy in our model, we formulate the reserve requirement as a linear function of technology parameter which is the source of systemic shocks,  $\tau_{t+1} = \tau_c + \tau E\theta_{t+1}$ .  $\tau_c$  is the constant parameter,  $\tau$  is the linear dependence to the state of economy and both parameters are nonnegative. We also try various functional forms and our simulation results show that linear function is the optimal form of macro prudential policy.

In this functional form, if the expected technology parameter is below its steady state level and so the prices, optimal reserve requirement level drops. In the opposite case, the reserve requirement level increases and prevent banks from taking excess risk. Rest of the solution is similar to constant reserve requirement case and first order condition of the problem requires;

$$(1-(\tau_c + \tau E\theta_{t+1}))q_{t+1}^s (\bar{n} - \mu \frac{(\tilde{n}_{t+1})^2 - (n^L)^2}{2(n^H - n^L)} + \mu \frac{\tilde{n}_{t+1}}{n^H - n^L} \frac{\pi_{t+1}^B}{\pi_{t+1}^{B'}}) = a_{t+1} \quad (2.27)$$

The parameters of optimal reserve requirement depend on the volatility of the shock in economy. When the volatility of systemic shock in the economy is above the threshold level, optimal parameters are  $\tau = 1 - \alpha$  and  $\tau_c = 0$  which is same as constant reserve requirement. If the volatility is below the threshold, both parameters are positive depending on the volatility level. Figure 2.3 displays the welfare comparison of constant reserve requirement, prudential reserve requirement and decentralized economy under different volatility of shocks. In Figure 2.3, it is obvious that prudential reserve requirement is pareto better than constant reserve requirement policy and decentralized economy. These results are justification of reserve requirement regulations. Moreover, it implies that difference between prudential approach and constant reserve requirement is significant when standard deviation of shocks is below the threshold, 0.6. In order to give an idea, most studies in the literature choose standard deviation of shocks between 0.13-0.3 in the empirical analysis, which implies the superiority of prudential approach according to our results.

### 2.6.3 Bailout financed by Wage Tax

During the crisis period, it is a common policy for governments to bail out the financial institutions using tax payers' money. In this section, we examine the long run welfare effects of bailout policy financed by tax on depositors' wage in a stochastic economy.

We revise our benchmark model by allowing the government to bail out  $x$  percent of the banks. Government puts wage tax,  $T_t$  to finance its bailout expenditures at time  $t$ .

$$(1-\eta)T_t w_t^d h_t = \left[ F(\tilde{n}_t) r_t^d d_t^s x - \int_{(1-x)\tilde{n}_t}^{\tilde{n}_t} q_t l_t n_t f(n) \right] \eta \quad (2.28)$$

Right side of the equation (2.28) is the government expenditure for bailouts. Equation (2.28) gives the required tax level,  $T_t$  at each period.

Bailout policy limits monitoring power of depositors over the banks. Since in case of defaults government pays the deposits, depositors don't put an extra risk spread on the deposit rates. Banks internalize such effects of bailout policy in their objective functions. Revised form of the bank's objective function is;

$$\max_{l_{t+1}} q_{t+1} l_{t+1} \bar{n} - \mu q_{t+1} l_{t+1} \int_{n_L}^{(1-x)\tilde{n}_{t+1}} n_{t+1} f(n) dn - a_{t+1} (l_{t+1} - w_{t+1}) + F(\tilde{n}_t) r_{t+1}^d d_{t+1}^s x - \int_{(1-x)\tilde{n}_{t+1}}^{\tilde{n}_{t+1}} q_{t+1} l_{t+1} n_{t+1} f(n)$$

Bank's first order condition becomes;

$$q_{t+1} (\bar{n} - (\mu(1-x)^2 - x^2) \frac{(\tilde{n}_{t+1})^2 - (n^L)^2}{2(n^H - n^L)} + (\mu(1-x)^2 - x^2) \frac{\tilde{n}_{t+1}}{n^H - n^L} \frac{\pi_{t+1}^B}{\pi_{t+1}^B}) = a_{t+1} \quad (2.29)$$

Notice that when there is no government bailout,  $x=0$ , equation (2.29) is same as the first order condition of decentralized economy given in equation (2.10). Figure 2.4 displays the comparison of welfare with decentralized economy. Banks are worse off in all states of the economy. Due to government guarantee and low cost of funding, banks takes excess risks and increase lending up to inefficient levels. High lending lowers the asset prices and bank profits shrinks in the next period. Thus default rates and bailout costs increases excessively in the

economy. Depositors pay high taxes to finance the bailout costs. Therefore, a pure bailout policy financed by wage tax which is applied to preserve economic stability makes the welfare of all agents worse off in the long run.

#### 2.6.4 Bailout Financed by Bank Tax

The main criticism about the bailout policies is to spend tax payers' money to pay costs of the banks' gambling. Especially ethical discussions stress that the bailout cost should be paid again by the banking system. But could the welfare of the economy improve if healthy banks pay the bailout costs? In this section, we answer this question by analyzing the welfare impacts of a bailout policy financed by the taxes on the healthy banks.

We revise our benchmark model by imposing government balanced budget condition at each period. It implies that;

$$q_t l_t \pi_t^B T_t = F(\tilde{n}_t) r_t^d d_t^s x - \int_{(1-x)\tilde{n}_t}^{\tilde{n}_t} q_t l_t n_t f(n) \quad (2.30)$$

where  $T_t$  is the tax on the profit of healthy banks. Right side of the equation is the bailout cost for  $x$  percent of the default banks.

Banks know the tax rate function but they don't internalize the effect of their leverage decision on the bailout tax. Similar to asset price discussion in the benchmark model, they think they are too small to affect the tax rate. The lack of such internalization kills the efficiency of the policy. Banks still ignore the cost of their decisions at the margin in their objective function. Banks' revised objective function in the model is;

$$\max_{l_{t+1}} q_{t+1} l_{t+1} \bar{n} - \mu q_{t+1} l_{t+1} \int_{n_L}^{(1-x)\tilde{n}_{t+1}} n_{t+1} f(n) dn - a_{t+1} (l_{t+1} - w_{t+1})$$

The relevant first order condition becomes;

$$q_{t+1} (\bar{n} - \mu(1-x)^2 \frac{(\tilde{n}_{t+1})^2 - (n^L)^2}{2(n^H - n^L)} + \mu(1-x)^2 \frac{\tilde{n}_{t+1}}{n^H - n^L} \frac{\pi_{t+1}^B}{\pi_{t+1}^B}) = a_{t+1} \quad (2.31)$$

Notice that equation (2.31) is same as equation (2.10) when bailout rate,  $x$  is zero. Figure 2.4 shows the welfare of the agents in the economy. Utilities of banks' shareholders are worse than their levels in decentralized equilibrium and utilities of depositors are just slightly better. The mechanism is similar to bailout policy with wage tax described in previous section. Banks don't internalize effects of their lending decisions on tax rates. They take excess risk as they think government pays bailout with a general profit tax. But since the general profit tax increases due to atomistic banks' excess risk taking, banks pay high taxes and banks' shareholders' utility is even lower than wage tax version. On the depositors' side, since their deposit income is guaranteed and bailout costs paid by healthy banks in the system, depositors will be slightly better off.

### 2.6.5 Combination of Policies: Prudential Reserve Requirement and Bailout

In this section, we analyze application of prudential reserve requirement and bailout policies at the same time. Bailout policy financed by profit tax on banks seems a plausible policy choice in terms of ethical norms. Banking system pays the cost of risky decisions and inefficiencies due to bankruptcy costs are prevented. But as we showed in previous section, banks don't internalize

effect of their decision on the tax rates and it induces excess risk taking. In order to limit banks' excess risk taking, we also apply prudential reserve requirement policy at the same time.

We revise our benchmark model using similar approach in reserve requirement and bailout sections. The tax equation is same as equation (2.30) in bailout section. After resolving the bank's problem by adding reserve requirement and bailouts, the new first order equation becomes;

$$(1-(\tau_c + \tau E\theta_{t+1}))q_{t+1}(\bar{n} - \mu(1-x)^2 \frac{(\tilde{n}_{t+1})^2 - (n^L)^2}{2(n^H - n^L)} + \mu(1-x)^2 \frac{\tilde{n}_{t+1}}{n^H - n^L} \frac{\pi_{t+1}^B}{\pi_{t+1}^B}) = a_{t+1} \quad (2.32)$$

Notice that when taxes and reserve requirements are zero in equation (2.32), it becomes same as decentralized economy's first order condition.

Figure 2.5 displays the results of the simulations for various volatilities of the systemic shock. In order to compare the policies, we choose reserve requirements in binding states such that all surpluses are transferred to banks. Considering shareholders' utility graph in Figure 2.5, it is obvious that combination of reserve requirement and bailout policies is better off than all other policies. In the previous section, we show that single bailout policy is even worse than decentralized economy. But when it is combined with prudential reserve requirement policy, we can prevent bankruptcy cost without inducing excess risk taking among the banks. Thus all agents' welfare improves. In addition, notice that as variance of shocks in the economy gets bigger, utility gains of risk averse depositors due to stability become more dominant than bankruptcy cost related gains under this policy. Therefore, as shown in Figure 2.5, when the



variance of shocks is high, welfare of the banks' shareholders converges to its levels in pure prudential policy.

### 2.6.6 Credit Default Swap (CDS)

In this section, we examine the effectiveness of credit default swaps against the shocks in the financial system. Credit default swap (CDS) is the one of the most popular insurance derivatives where the seller of CDS compensates the buyer in the event of loan default. We define the CDS such that buyer banks are compensated when the returns from their loans are below a threshold level. We analyze two different designs of insurance system.

#### 2.6.6.1 CDS issued under Government Protection

In this part, we design the derivatives such that sellers of CDS compensate the buyer banks when their loan returns are below the default threshold level. Compensation amount is the difference between realized return and threshold level. We assume that these CDSs are sold by only insurance agents. In addition, these insurance agents are bailed out by government when necessary. Therefore, banks have implicit bailout guarantee from the government and this is similar to bailout policies considered in previous sections.

When the CDS are introduced to the benchmark model, objective functions of the banks in the model become;

$$\max_{l_{t+1}} \int_{\tilde{n}_{t+1}}^{n_H} (q_{t+1} l_{t+1} n_{t+1} - a_{t+1} d_{t+1}) f(n) dn - C_{t+1} l_{t+1} \quad (2.33)$$

where  $C_{t+1}$  is the fix cost of CDS. Banks benefit from CDS insurance since depositors don't ask additional risk spread in deposit interest rates. Banks' funding cost is lower than decentralized economy and it is independent of their leverage decisions. Since there is informational asymmetry, insurance agents can't observe the exact level of risk that banks are taking. In addition, government insurance kills any monitoring incentive of insurance agents. Insurance agents' only goal is to sell as much CDSs as possible. In such a design, banks want to lend as much as possible as they don't bear the downside risks. The implications of such a system is similar to pure bailout policies mentioned in previous sections. Welfare in economy deteriorates due to inefficiently high lending.

#### ***2.6.6.2 CDS issued by Private Banks***

In the previous set up, banks don't bear the cost of excess risk. Therefore, in this section, we change the design of the insurance system such that both buyers and sellers of CDSs are the banks and there is no insurance agency or government protection. Banks insure themselves by exchanging CDS with every other bank. When a bank defaults, it is compensated up to the average level of loan returns in system. It is similar to creating pool for revenues of the banks and distributing these revenues to the banks evenly. Therefore idiosyncratic uncertainty disappears and each bank guarantees average level of return,  $\bar{n}$ .

If the benchmark model is revised, banks' objective function will be;

$$\max_{l_{t+1}} \int_{\bar{\theta}_{t+1}}^{\theta_H} (q_{t+1} l_{t+1} \bar{n}_{t+1} - r_{t+1}^d d_{t+1}) f(\theta) d\theta + C_{-i}(L_{t+1}) - c_i(l_{t+1}) \quad (2.34)$$

where  $C_{-i}(L_{t+1})$  is the revenue from CDS sold to all other banks given the aggregate lending  $L_{t+1}$ .  $c_i(l_{t+1})$  is the cost of buying CDS from other banks when the bank's own lending is at  $l_{t+1}$ .  $\theta$  is the systemic shock in the economy. Since CDS is the price of the default risk, its cost is equal to the expected loss in next period. Cost of CDS is;

$$c_i(l_{t+1}) = \int_{\theta_{t+1}}^{\theta_H} (r_{t+1}^d d_{t+1} - q_{t+1} l_{t+1} \bar{n}_{t+1}) f(\theta) d\theta$$

Bank's objective function in equation (2.34) is rewritten as:

$$\max_{l_{t+1}} \int_{\theta_L}^{\theta_H} (q_{t+1} l_{t+1} \bar{n}_{t+1} - r_{t+1}^d d_{t+1}) f(n) dn + C_{-i}(L_{t+1}) \quad (2.35)$$

Notice that, when banks exchange CDS, they implicitly pay all their liabilities even in the default. Therefore, limited liability situation is not valid anymore. This is the cost of getting rid of idiosyncratic uncertainty in their returns. Banks' possible losses in case of defaults are reflected in the price of CDSs they have bought. Thus, they implicitly internalize all their liabilities in their objective function. Therefore, their lending becomes lower compared to decentralized equilibrium. Such an insurance mechanism improves the welfare similar to reserve requirement policies.

CDSs improve the welfare under the assumption that systemic shocks are not big enough. However such an improvement comes with tail risk for the whole economy. If the systemic shock is big enough, asset prices can drop to a level where banks' total revenues can't pay their total debt. In such a case, defaults become contagious due to CDS protection and all the banks

default in the economy. But if banks don't exchange CDS, even if the systemic shock is big, the healthy banks with higher asset returns will not default. Therefore, benefits of CDSs are limited to the size of the shocks. Crises might be contagious due to CDS linkage and the financial system might be under risk. Contagious effect of CDSs is in line with the result of Stiglitz (2010) paper where he proves that a full financial integration might not be desirable due to contagious effects. Stiglitz (2010) paper shows that when the crisis in one country is big enough, it can be contagious to other countries of the union which are obliged to send aid. In this section, we show that CDSs create a similar linkage between banks and when the shocks are big enough, banks are forced to default because of these CDS exposure even if they are healthy.

### **2.6.7 Sensitivity Analysis**

In previous sections, we present our policy analysis under various levels of variance of systemic shock which is one of the key parameters in the model. In this section, we examine the sensitivity of our results to other parameters of the model. Table 2.1 to Table 2.4 show how the welfare of agents and leverage in the economy vary with parameters under different policy alternatives. Table 2.1 and Table 2.2 present the sensitivity results when the systemic price volatility is low. Table 2.3 and Table 2.4 present the sensitivity results when volatility is high.

In general, sensitivity analysis shows that our results and policy comparisons in previous sections are robust to the choice of parameters. Prudential reserve requirement has superiority over other regulations. Moreover, under reserve requirement, banks have lower leverage compared to other policies and these results are robust to parameter choices.

*Output elasticity of capital ( $\alpha$ ):* A decrease in capital share of consumption good production implies a decrease in asset prices and increase in depositors' wages. Therefore, depositors' welfare increases in all policy alternatives and in decentralized model. Lower asset prices reduce banks profitability, and thus, banks' leverage declines in all policies except bailout with bank tax scenario.

*Output elasticity of shareholders' labor ( $\alpha^H$ ):* An increase in the elasticity of labor for shareholders implies an increase in their wages and decrease in depositors' wages. It is similar to a transfer from depositors to banks' shareholders. Therefore, shareholders' welfare increases, while depositors' welfare drops compared to baseline parameterization in all policy alternatives.

*Discount Factor ( $\beta$ ):* A decrease in the discount factor leads to increase in risk free interest rates. Banks should offer higher deposit rates to convince depositors to save. As a result, higher deposit earnings improve depositors' welfare. On the other hand, higher funding cost lowers banks' profits. Moreover, shareholders prefer consumption instead of reinvesting and supplying capital to the banks. Thus both banks' initial capital and loans decline in the economy, but leverage doesn't change significantly.

*Persistence of Productivity ( $\rho$ ):* A decrease in persistence of shocks leads to decline in predictable component of the next period prices. Therefore, risk averse depositors cut their funding to banks and leverage drops. Risk averse depositors benefit from robust banking system due to less volatility in their deposit income.

*Risk Aversion of Depositors ( $\sigma$ ):* An increase in risk aversion implies a higher precautionary savings. Funding costs of banks declines and their profitability increases. They take higher risks

and raise the leverage. If depositors are assumed risk neutral, depositors will cut their savings and banks will struggle to finance their lending. In such a case, leverage in the economy and banks' profits drop.

*Bankruptcy Cost ( $\mu$ ):* Bankruptcy cost is an important determinant of the risk spread on deposit interest rates. A decline in bankruptcy costs leads to a decrease in deposit interest rates. Thus banks' risk appetite increases and they raise leverage. However higher leverage depresses the asset prices next period and weakens banks' profits. On the other hand, depositors get rid of paying high monitoring costs and their welfare increases.

*Shareholders' Discount Factor ( $\gamma$ ):* Shareholders' discount factor is important for allocation of their income between consumption and reinvestment on the bank. A decrease in the discount factor leads to a decrease in reinvestment on the banks as shareholders perceive future less valuable. Decreasing capital supply raises banks leverage and default risks. Therefore, depositors ask higher spreads and banks' profits decline. Since banks become more sensitive to systemic shocks, volatility of deposit income increases and depositors' welfare decreases.

*Banks Loan Returns ( $n^H$ ):* A decline in the upper level of loan returns deteriorates bank's profitability. Their balance sheet weakens and default risk increases. As banks become more sensitive to the shocks, volatility in their deposit payments increases and depositors' welfare declines.

*Agents' Population ( $\eta$ ):* A decrease in shareholders' ratio of population implies higher supply of deposits compared to the demand. Deposit rates decline and banks increase leverage.

Banks benefit from cheap funding and their profits increase. On the other hand, depositors saving income and welfare decline.

## **2.7 Conclusion**

This paper investigates the effects of asset price externalities on welfare of the banks and depositors in a model with the bank defaults and bankruptcy costs at equilibrium. Decentralized banks do not internalize effects of their lending decisions on distribution of asset prices and such pecuniary externalities leads banks to take excess risk by over-borrowing. High leverage induces excess supply of assets and depresses the asset prices in future periods. This makes the banks vulnerable to systemic shocks and prevents their capital accumulation. In addition, increasing volatility in deposit repayments and bankruptcy costs deteriorate depositors' utilities. We show that even a restricted social planner can increase robustness of banks against systemic shocks, and improve the social welfare by reducing the leverage ex-ante.

The key contribution of this paper is the analysis of such pecuniary externality under variety of regulations and bailout policies. We show that generic inefficiencies of competitive equilibrium exist in financial markets. Moreover, bankruptcy costs and systemic price shocks amplify these externalities. We find that mitigating such distortions is possible with regulations that limit the risk appetite of banks depending on the state of economy. Our main conclusion is that there is much to gain from introducing reserve requirement in macroprudential form.

Moreover, combining this macroprudential policy with bailout guarantee also prevents bankruptcy costs and improves the social welfare more.

On the policy side, we consider different forms of bailout policies. We show that in general, pure bailout policies raise the risk level in the economy. Bailout guarantees limit depositors' monitoring power and they ask lower risk spreads for their deposits. Banks increase their leverage since their funding costs decrease. Excess risk taking depresses asset prices and profitability of banking sector in the future periods. Thus banks become vulnerable to systemic shocks.

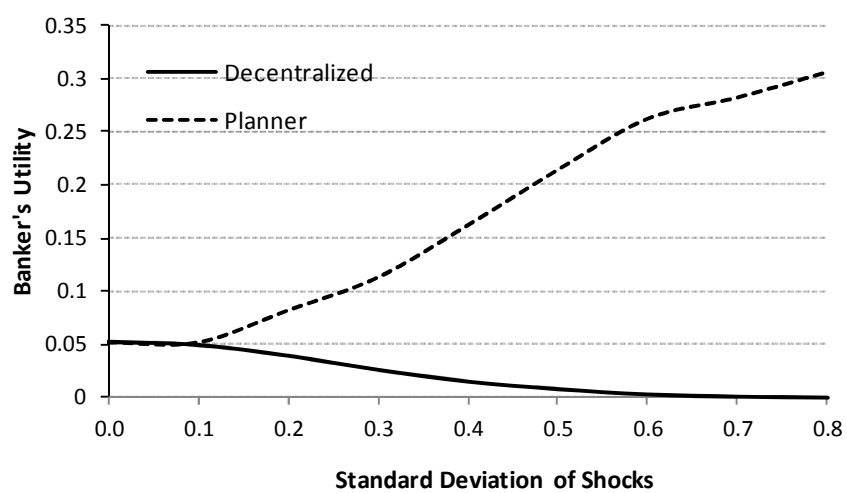
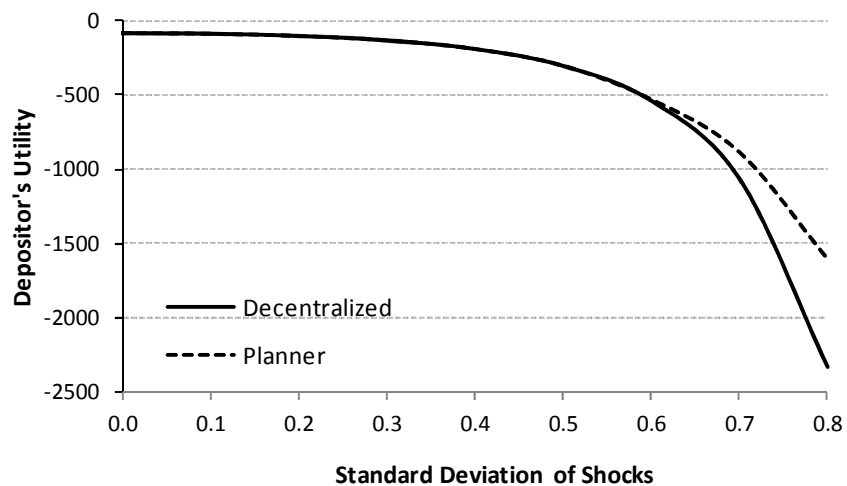
Using the similar framework, we also examine the policies that restrict lending in the economy. We find that policies with predetermined threshold levels for the leverage like the Basel criteria are not efficient. Such policy measures don't help banks to internalize the costs of the risks. Instead, these measures define just an upper level for the choice of leverage. On the other hand, we show that bank can internalize the effects of their lending decisions with reserve requirements. Furthermore, prudential reserve requirement improves the social welfare more as its elastic structure provides a buffer against systemic shocks. Banks perceive the cost of lending higher during good states of the economy and limit their lending. In the bad states, low reserve requirements decrease the cost of lending for the banks and banks increase lending. Such a mechanism increases resistance of financial system to systemic shocks. Risk averse depositors also benefit from more stable economy.

As an alternative policy choice, we combine bailout policy with prudential reserve requirement. Our results show that such a combination is better than other single policies.

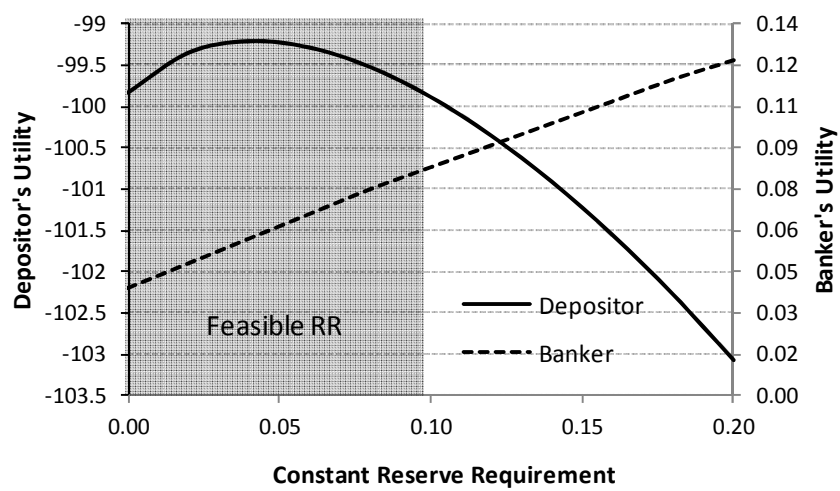


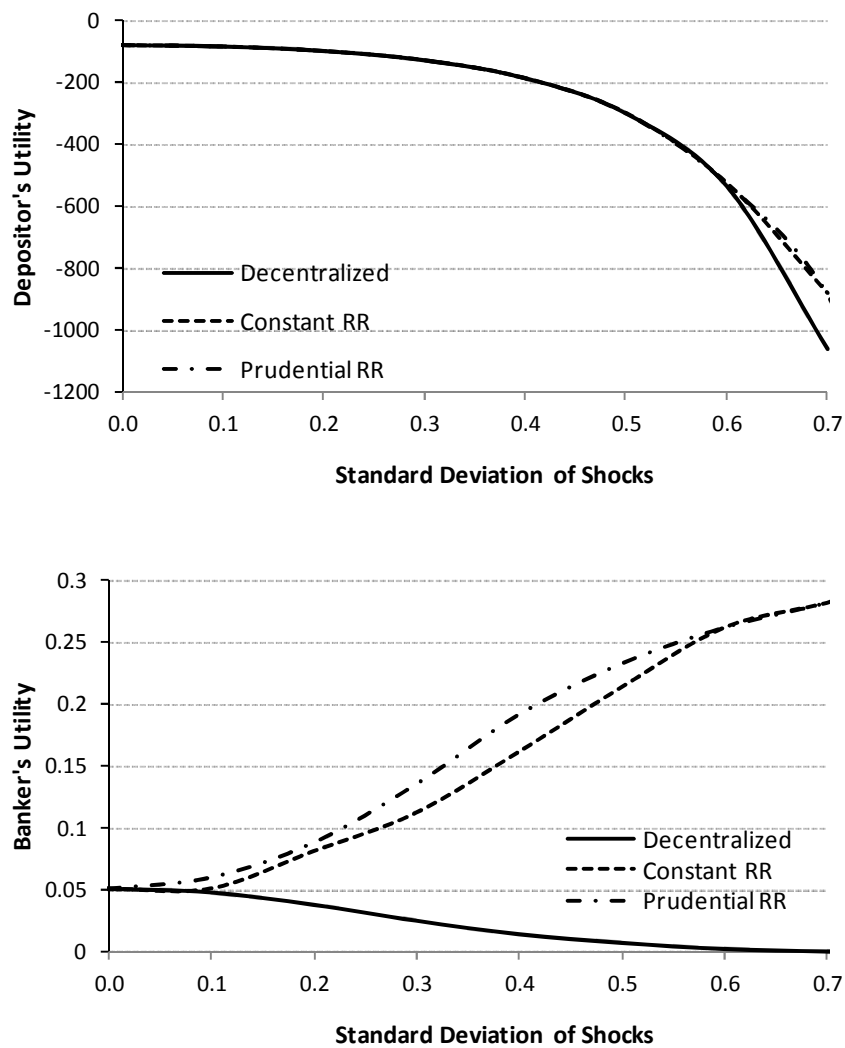
Bailout policy prevents the bankruptcy costs in the economy. However it causes moral hazard problem among the banks. Prudential reserve requirement increases cost of lending and helps banks to internalize costs of their leverage decisions. This prevents the moral hazard problems in financial sector. Therefore, when we apply bailout policy and prudential reserve requirement at the same time, social welfare improves.

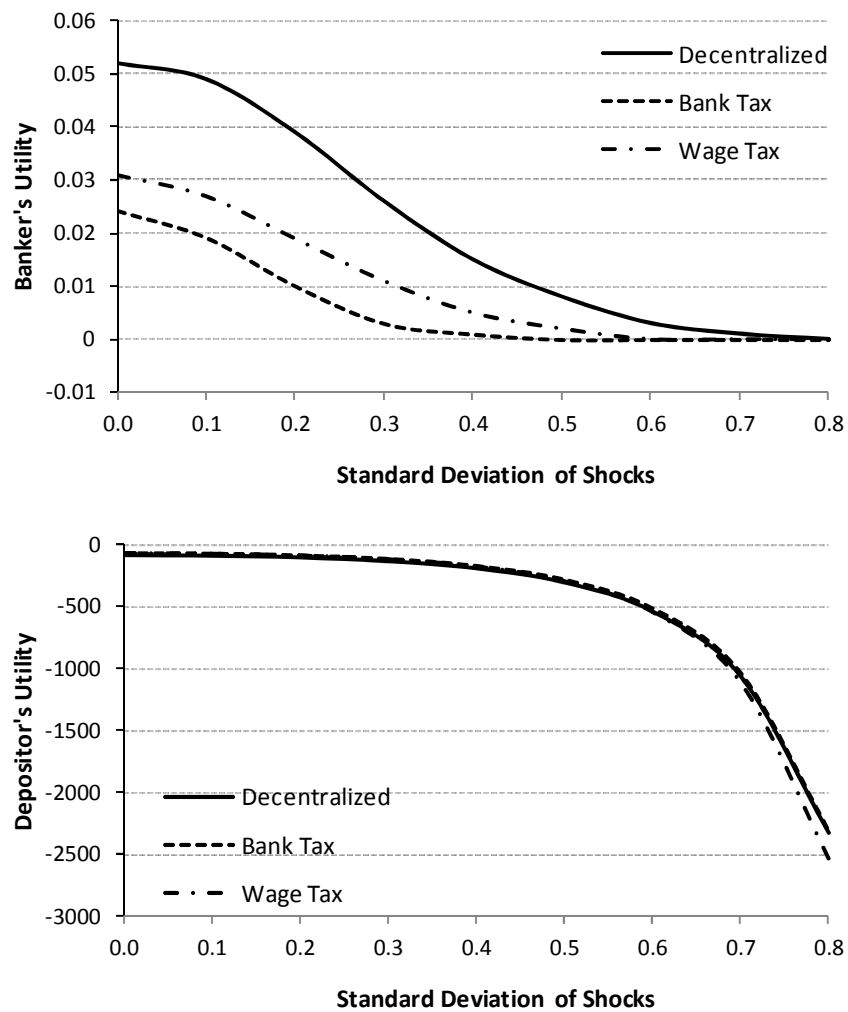
In the last step, we extend our analysis by adding credit default swap (CDS) into our model. We show that CDSs with implicit government guarantee and bailout policies have similar effects. In both cases, banks don't internalize the cost of excess lending and they increase the leverage. Therefore, agents' welfare deteriorates. On the other hand, we show that when CDSs are issued by private banks without government guarantee, banks implicitly internalize cost of their excess lending due to higher prices of CDSs. Banks share the burden of idiosyncratic risk among themselves and such a mechanism limits their exposure to the uncertainties. Thus social welfare improves under the condition that systemic shocks are not big enough. However if the systemic shocks are big enough, such a CDS linkage can have contagious effects. Even banks are healthy enough for such shocks; they might default due to CDS exposure from weaker banks.

**Figure 2.1: Planner's Problem vs Decentralized Equilibrium**

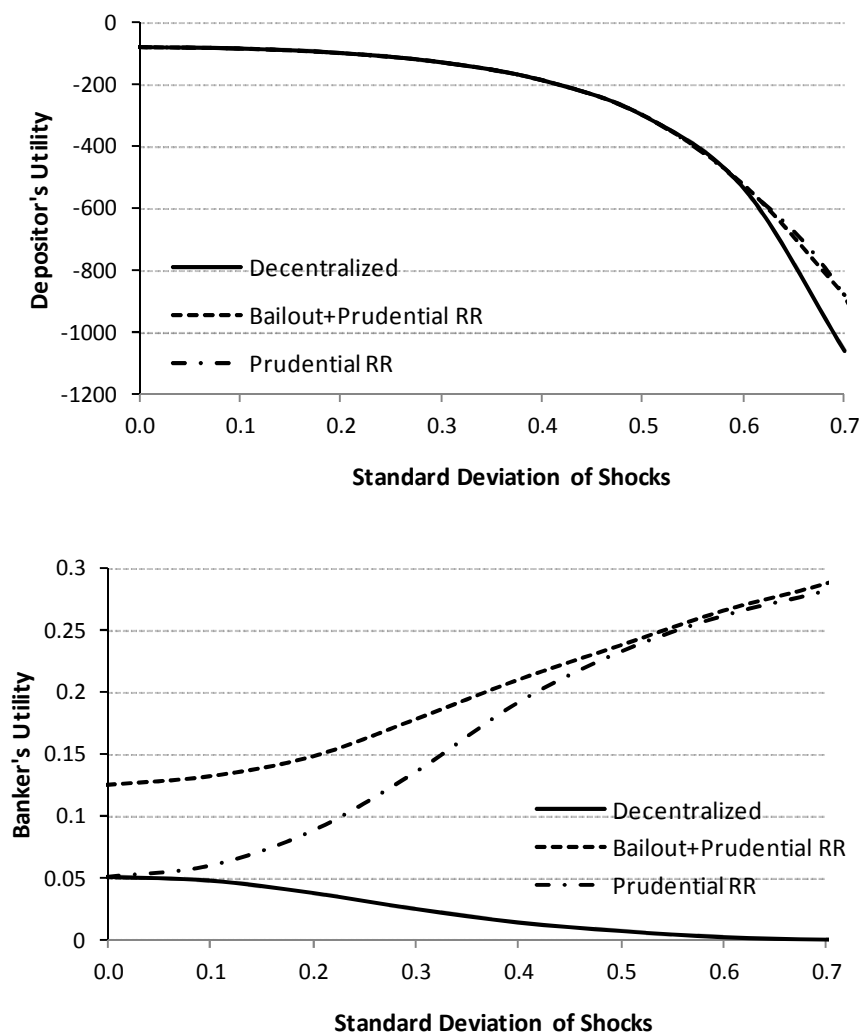
**Figure 2.2: Feasible Reserve Requirement Level ( $\sigma=0.2$ )**



**Figure 2.3: Utility of Agents: Prudential RR vs Constant RR**

**Figure 2.4: Utility of Agents with Bailout Policies: Bank Tax vs Wage Tax**

**Figure 2.5: Utility of Agents: Prudential RR vs Bailout and Prudential RR**



**Table 2.1: Sensitivity Analysis (standard deviation=0.2)**

	Decentralized				Constant RR				Prudential RR		
	Utility				Utility				Utility		
	Dep.	Bank	Lev.		Dep.	Banker	Lev.		Dep.	Banker	Lev.
Baseline	-99.8	0.04	22.7		-99.8	0.08	10.7		-99.8	0.09	9.5
$\alpha=0.3$	-83.2	0.04	22.6		-83.2	0.08	10.6		-83.2	0.09	9.5
$\alpha^H=0.1$	-127.8	0.15	22.6		-127.8	0.19	10.6		-127.9	0.20	9.5
$\beta=0.9$	-38.4	0.04	22.6		-38.4	0.08	10.6		-38.5	0.09	9.5
$\rho=0.7$	-87.3	0.04	22.0		-87.3	0.08	10.4		-87.5	0.09	9.6
$\sigma=0$ (risk neutral)	9.4	0.04	22.5		9.3	0.08	10.4		9.2	0.09	9.2
$\sigma=5$	-16199.7	0.05	27.7		-15417.5	0.10	14.8		-14122.3	0.11	13.5
$\mu=0.1$	-90.9	0.01	171.6		-91.8	0.06	17.9		-91.8	0.07	15.6
$\gamma=0.2$	-101.2	0.03	39.9		-101.0	0.07	18.1		-101.0	0.07	16.1
$n^H=1.5$	-119.5	0.01	22.9		-118.3	0.04	10.2		-118.1	0.04	9.4
$\eta=0.1$	-102.4	0.16	24.1		-102.6	0.33	11.6		-102.0	0.34	11.2

Notes: Baseline parameters are;  $\alpha=0.35$ ,  $\alpha^H=0.01$ ,  $\beta=0.96$ ,  $\rho=0.9$ ,  $\sigma=2$ ,  $\mu=0.36$ ,  $\gamma=0.3$ ,  $n^H=2$ ,  $\eta=0.33$

**Table 2.2: Sensitivity Analysis (standard deviation=0.2)**

	Decentralized				Bailout: Bank Tax				Bailout: Wage Tax		
	Utility				Utility				Utility		
	Dep.	Bank	Lev.		Dep.	Banker	Lev.		Dep.	Banker	Lev.
Baseline	-99.8	0.04	22.7		-94.6	0.01	90.9		-96.0	0.02	61.8
$\alpha=0.3$	-83.2	0.04	22.6		-79.7	0.01	91.5		-80.6	0.02	61.5
$\alpha^H=0.1$	-127.8	0.15	22.6		-121.2	0.13	92.2		-123.3	0.14	61.8
$\beta=0.9$	-38.4	0.04	22.6		-36.3	0.01	92.2		-36.8	0.02	61.8
$\rho=0.7$	-87.3	0.04	22.0		-82.7	0.01	88.4		-83.9	0.02	59.9
$\sigma=0$ (risk neutral)	9.4	0.04	22.5		9.9	0.01	90.1		9.8	0.02	61.3
$\sigma=5$	-16199.7	0.05	27.7		-13374.1	0.01	112.3		-14433.8	0.03	76.0
$\mu=0.1$	-90.9	0.01	171.6		-87.2	0.00	0.0		-91.9	0.02	168.1
$\gamma=0.2$	-101.2	0.03	39.9		-95.2	0.01	201.1		-96.8	0.02	112.5
$n^H=1.5$	-119.5	0.01	22.9		-114.1	0.00	116.5		-115.9	0.00	67.3
$\eta=0.1$	-102.4	0.16	24.1		-96.8	0.05	63.1		-98.2	0.09	60.7

Note: Baseline parameters are;  $\alpha=0.35$ ,  $\alpha^H=0.01$ ,  $\beta=0.96$ ,  $\rho=0.9$ ,  $\sigma=2$ ,  $\mu=0.36$ ,  $\gamma=0.3$ ,  $n^H=2$ ,  $\eta=0.33$



**Table 2.3: Sensitivity Analysis (standard deviation=0.6)**

	Decentralized				Constant RR				Prudential RR		
	Utility		Lev.		Utility		Lev.		Utility		Lev.
	Dep.	Bank			Dep.	Banker			Dep.	Banker	
Baseline	-535	0.003	28.8		-526	0.262	3.5		-526	0.262	3.5
$\alpha=0.3$	-362	0.004	27.2		-360	0.242	3.4		-360	0.242	3.4
$\alpha^H=0.1$	-686	0.104	28.2		-674	0.363	3.5		-674	0.363	3.5
$\beta=0.9$	-137	0.003	28.1		-133	0.253	3.5		-133	0.253	3.5
$\rho=0.7$	-159	0.004	22.0		-161	0.262	3.1		-161	0.262	3.1
$\sigma=0$ (risk neutral)	45.2	0.00	26.8		33.5	0.2	1.2		33.5	0.2	1.2
$\sigma=5$	-2E+15	0.048	172.3		-4E+13	0.483	1264.5		-4E+13	0.483	1264.5
$\mu=0.1$	-515	0.000	62.6		-523	0.264	3.5		-523	0.264	3.5
$\gamma=0.2$	-568	0.002	48.2		-558	0.234	4.4		-558	0.234	4.4
$n^H=1.5$	-782	0.000	31.1		-661	0.145	2.3		-661	0.145	2.3
$\eta=0.1$	-559	0.010	50.1		-524	1.137	4.0		-524	1.137	4.0

Note: Baseline parameters are;  $\alpha=0.35$ ,  $\alpha^H=0.01$ ,  $\beta=0.96$ ,  $\rho=0.9$ ,  $\sigma=2$ ,  $\mu=0.36$ ,  $\gamma=0.3$ ,  $n^H=2$ ,  $\eta=0.33$

**Table 2.4: Sensitivity Analysis (standard deviation=0.6)**

	Decentralized				Bailout: Bank Tax				Bailout:Wage Tax		
	Utility		Lev.		Utility		Lev.		Utility		Lev.
	Dep.	Bank			Dep.	Banker			Dep.	Banker	
Baseline	-535	0.003	28.8		-520	0.000	24.4		-551	0.000	60.3
$\alpha=0.3$	-362	0.004	27.2		-354	0.000	26.1		-371	0.001	57.7
$\alpha^H=0.1$	-686	0.104	28.2		-667	0.101	27.7		-714	0.109	60.6
$\beta=0.9$	-137	0.003	28.1		-133	0.000	27.7		-141	0.000	60.4
$\rho=0.7$	-159	0.004	22.0		-155	0.000	19.1		-164	0.001	45.5
$\sigma=0$ (risk neutral)	45.2	0.00	26.8		47.4	0.00	22.7		45.8	0.00	56.2
$\sigma=5$	-2E+15	0.048	172.3		-2E+15	0.000	164.6		-3E+15	0.006	389.7
$\mu=0.1$	-515	0.000	62.6		-430.33	0.000	0.0		-658	0.012	0.0
$\gamma=0.2$	-568	0.002	48.2		-544.74	0.000	7.3		-580	0.000	97.9
$n^H=1.5$	-782	0.000	31.1		-808	0.000	227.7		-872	0.000	129.8
$\eta=0.1$	-559	0.010	50.1		-532	0.000	0.9		-564	0.002	51.0

Note: Baseline parameters are;  $\alpha=0.35$ ,  $\alpha^H=0.01$ ,  $\beta=0.96$ ,  $\rho=0.9$ ,  $\sigma=2$ ,  $\mu=0.36$ ,  $\gamma=0.3$ ,  $n^H=2$ ,  $\eta=0.33$

# **Chapter 3**

## **Leverage and Default Risk of Financial Intermediaries under Asymmetric Information**

### **3.1 Introduction**

Financial intermediaries such as commercial banks, investment banks, hedge funds, etc. are crucial institutions in the effort to decrease inefficiency and risk in economy. The financial crisis in 2008, exacerbated by the excessive risk-taking of banks and their subsequent bailouts, stressed

the importance of banking regulations. Before 2008 global crisis, financial intermediaries innovated financial tools like derivatives to go around limitations of banking regulations. These innovations caused asymmetric information problems between depositors and financial intermediaries. Depositors lost their monitoring power over financial intermediaries. Thus financial intermediaries were able to choose high risk levels without paying any extra cost. As a result of such a financial market design without strict regulations, we faced with a global financial crisis. Therefore, to prevent possible future financial crises, it is essential to re-examine the effects of asymmetric information and the design of new regulations like stress tests.

This paper investigates effects of depositors' monitoring power on the financial institutions' leverage decisions under different levels of asymmetric information. In addition, we also analyze incentives of government to prevent asymmetric information with regulations such as stress tests. The key contribution of the paper is to show analytically excessive risk taking behaviors of banks under different levels of imperfect monitoring in a model with bankruptcy costs.

This paper develops a single period model with three types of agents; depositors, borrowers and bankers. Definition of banks in the model is wider and it includes financial intermediaries. Depositors invest in banks by insuring themselves against the default risk. They have standard debt contract with banks as defined in Williamson (1986). Verification of the state is costly as in Townsend (1979) and asymmetric information is ex-post on the realization of banks' return. In the benchmark model, depositors can monitor balance sheet of banking system and estimate the

default probability. In the extension of the benchmark model, different levels of asymmetric information and noises are introduced into depositors' monitoring.

Borrowers are the agents without endowment. Their production depends on the inputs from borrowing and it has stochastic technology parameter. This stochastic technology parameter is the source of uncertainty in the economy. We assume that banks have skills to monitor their borrowers, and thus, they have perfect information about the firms. Therefore, the debt contract is perfectly state contingent or equity debt. It is similar to assume that bankers own the firm against their debt.

Bankers are the owners of the banks. They have endowments that could be used in banks' operations as capital. Bankers' problem is to choose the optimal deposit level for the banks. While deciding on the leverage ratio they also determine banks' default probability.

Banks are at the center of the model. Direct lending of depositors to borrowers is out of the equilibrium as banks have special skills to monitor borrowers and they prevent duplication of monitoring costs. Banks increase the efficiency in the economy but they are also fragile institutions. They are balance sheet constrained and they face with two types of risks due to this structure. The risks can be either from depositors such as bank run because of maturity mismatch or from borrowers like default of borrowers. The benchmark model is focused on the risk from borrowers.

After analyzing the benchmark model, we add different levels of information asymmetries into the model. We classify banks into two groups with respect to their loan return abilities; good

and bad. Level of asymmetric information depends on information set of the depositors about banks' types. Banks ex-ante know their types but depositors don't know. However, depositors might have information or beliefs regarding the distributions of loan returns of bank types. We show that if the bad type banks' ratio is high in the economy with asymmetric information, depositors couldn't detect these banks and there will be excess risk taking like the pre-crisis period of 2008 financial crisis. On the other hand, if good type banks' ratio is high, depositors couldn't reward the good type banks and aggregate lending will be below its potential due to lack of reliability to banks.

After describing the analytic solutions of asymmetric information problems, we evaluate the efficiency and reliability of stress tests. Stress tests are used to prevent asymmetric information in financial markets. However, there are still questions about the incentives of governments to announce the real type of the banks. This paper analyzes government incentives under asymmetric information and maturity risk. We proved that under certain conditions, covering up stress tests' results is the optimum decision for the governments.

The model in this paper builds on the literature of macroeconomic models with financial intermediaries. Literature started with finite period models, focusing on the relationship between lender and entrepreneur. Townsend (1979) introduced costly monitoring problem. Then literature was separated into two different paths. First branch (Stiglitz and Weiss, 1981 and Gale and Hellwig, 1985) concentrated on credit rationing in equilibrium while the other branch (Boyd and Prescott, 1986; Diamond, 1984 and Williamson, 1986) tried to explain role and existence of

financial intermediaries. Diamond (1984) was the first study explaining the existence of banks. In his paper, banks are defined as delegated monitors and monitoring decisions are ex-ante. In the studies of Townsend (1979), Gale and Hellwig (1985) and Williamson (1986); agents are informed asymmetrically on the realization of projects and monitoring decisions are ex-post.

Following work in the literature integrated specific financial contracts into the general equilibrium models with infinite periods. Carlstrom and Fuerst (1997) used information asymmetries in a general equilibrium framework by defining heterogeneous agents, specifically lenders and borrowers. Kiyotaki and Moore (1997) presented a similar approach with the financial accelerator mechanism which limits the borrowers' credit amount depending on their collateralized assets. Bernanke et al. (1999) added credit market frictions and asymmetric information into the dynamic general equilibrium model. Greenwald and Stiglitz (2003) explained in their book possible ways of setting up general equilibrium mechanism with a banking system.

In this paper, different than the literature, we examine the effects of information asymmetries at various levels in the financial system with default costs. We evaluate the impacts of imperfect monitoring on banks' behaviors using a single period model with bankruptcy costs. Moreover, we analyze government incentives to carry out unbiased stress tests in the similar analytical framework.

The rest of the paper is organized as follows. Section 2 describes the benchmark model and provides analytical solution. Section 3 presents different level of information asymmetries and

describes their effect on the economy. Section 4 examines the reliability of stress tests by analyzing the government's incentives and section 5 concludes.

## 3.2 Benchmark Model

### 3.2.1 Depositors

Depositors are the financier of the banks. They have initial endowment,  $W^d$  and their utility function is risk neutral. Depositors have standard debt contract with the banks (Williamson, 1984). They get interest rate  $r^d$ , for each unit of deposit when banks do not default. When bank defaults, depositors get all the remaining capital of the bank and pay the bankruptcy cost. We define bankruptcy costs as monitoring and liquidation costs. In order to simplify problem, we assume depositors' bankruptcy costs are large enough that depositors get nothing when the bank defaults.

Depositors ask additional spread over risk free rate to insure themselves against the banks' default risks. The participation constraint for the depositor is given by;

$$r^d(1 - F(\tilde{n})) = a \quad (3.1)$$

where  $F(\tilde{n})$  is the bank's default probability<sup>1</sup> and  $a$  is the risk free rate. Depositors can observe banks' leverage ratio, and thus, they can estimate their default probability.

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<sup>1</sup> Derivation of the default probability is explained in the banks' section.



### 3.2.2 Borrowers

Borrowers' initial endowment is zero. In order to produce, they need to borrow from banks. Their technology is assumed to be linear,  $y=nL$ . They use bank loans,  $L$ , as capital in their production. The productivity factor,  $n$  is continuously distributed over the interval  $[n^L, n^H] \subseteq \mathbb{R}^+$  with uniform density function,  $f(n)$ . Banks have perfect monitoring skills over borrowers. The contract between borrowers and the banks is assumed perfectly state contingent debt contract similar to issuing the equity for their debt. Borrowers transfer all their revenues to the banks due to the specification of the contract. Banks' return from borrowers is given by:

$$\pi = \int_{n^L}^{n^H} nL f(n) dn \quad (3.2)$$

### 3.2.3 Banks

Banks are owned by the bankers who are risk neutral agents. Bankers have initial capital,  $W$  that is used as bank's capital. In the single period model, bankers' problem turns into the profit maximization problem of banks. In case of bank default, there is limited liability and the bankers lost only their shareholder rights. They don't bear any additional bankruptcy cost in defaults.

Banks are at center of the model as they are receiving deposits from the depositors and lending to the borrowers. Direct borrowing between depositors and lenders is out of the equilibrium because in direct lending depositors should pay monitoring cost for each borrower

and this duplicates their monitoring costs. In addition, banks have better monitoring power over borrowers. Therefore, banks exist in the equilibrium.

Banks increase efficiency in the economy but they are fragile institutions. They are balance sheet constrained and they should be solvent at all times. They face with two types of liquidity risks; maturity mismatch problem from depositors' side and default risk from borrowers' side. This study focused on the default risk from borrowers' side. However, at the end of the paper, we also consider maturity risk. We define default risk like systemic risk. We assume borrowers' returns are stochastic but perfectly correlated. It is similar to the relation between weather conditions and harvest. Weather conditions are stochastic. But, if the weather is bad, all farms' harvests will decline. It is a systemic risk that prevents risk distribution ability of farmers like banks as it is observed in most of the crisis.

Banks will default if they are insolvent. In order to avoid from defaults, their liabilities (deposit repayments) should be higher than the loan returns. Default constraint is given by:

$$nL - r^d D \geq 0 \quad (3.3)$$

Banks can put some portion of their deposits aside, referred as cash reserves. Cost of the cash reserves is the deposit rate,  $r^d$  and their return is 1. Since we ignore maturity-mismatching

problem, holding cash increases the default probability of the banks. Therefore, optimal level of cash reserves is zero in our model<sup>2</sup>. Thus banks lend all of the deposits and capital,  $W+D=L$ .

There is a threshold level of productivity factor,  $\tilde{n}$  that makes the default constraint (3.3) binding.  $\tilde{n}$  is given by:

$$\tilde{n} = \frac{r^d D}{D+W} \quad (3.4)$$

If the realized productivity of borrowers is lower than the threshold level, banks' revenues will not be adequate to pay liabilities and banks will default. Banks' default probability is the cumulative distribution function of productivity at  $\tilde{n}$ ,  $F(\tilde{n})$ . Using the uniform distribution assumption, banks' default probability can be simplified to:

$$F(\tilde{n}) = \frac{1}{n^H - n^L} \left( \frac{r^d D}{D+W} - n^L \right) \quad (3.5)$$

Banks' default rate is increasing with their deposit level and decreasing with their initial capital. Higher the deposits imply higher the loans and banks' leverage. However, if the initial capital rises, it will pull down the leverage ratio and banks' structure will be less risky.

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<sup>2</sup> In most developed countries and US, there is no requirement for holding cash reserves. In US, cash reserve to asset ratio is 3 percent on average. In other countries like China, it is mostly used as a tool to control speed of money and inflation.

### 3.2.4 Representation of Problem

Banks maximize their profits subject to depositors' participation constraint. Banks' optimization problem is given by:

$$\begin{aligned} \max_D \int_{\tilde{n}}^{n^H} (n(D+W)-r^d D)f(n)dn \\ \text{s.t.} \\ r^d(1-F(\tilde{n})) \geq a \end{aligned} \quad (3.6)$$

Depositors' participation constraint is always binding. Banks' participation constraint is given by:

$$\int_{\tilde{n}}^{n^H} (n(D+W)-r^d D)f(n)dn \geq W$$

We assume returns are attractive enough to convince bankers to borrow. Therefore, banks' participation constraint is always satisfied and it can be ignored while solving the optimization problem.

Assuming uniform distribution of returns, the banker's first-order condition requires:

$$\frac{dr^d}{dD} D + r^d - \frac{n^H + \tilde{n}}{2} = 0 \quad (3.7)$$

### 3.2.5 Solution of Model

In order to have a solution for the optimization problem, there should be an interest rate that convinces depositors to put their money into the banks. Solution of the depositors' participation constraint provides the conditions for existence of such an interest rate.

**Proposition 1:** Participation constraint determines the feasible set of deposits in the bank's optimization problem. Solution exists if there is an interest rate which satisfies depositor's participation constraint for the given deposit demand of bank.

- (i) If  $n^H \geq \bar{n}$ , for every  $D$ , there exists a feasible deposit interest rate.
- (ii) If  $n^H < \bar{n}$ , there exists an interest rate if and only if  $D \in [0, \hat{D}]$

$$\text{where } \hat{D} = \frac{(n^H)^2 W}{4a(n^H - n^L) - (n^H)^2} \text{ and } \bar{n} = 2 \left[ a + \sqrt{a(a - n^L)} \right]$$

Proof: See Appendix E.

Part (i) of proposition 1 implies that if the bank's returns are high enough, depositors will accept an interest rate for each level of bank's borrowing. When the bank's returns are not high enough and the bank demands high deposits as given in part (ii) of proposition 1, depositors don't want to lend at any interest rate due the high leverage levels and high default risk. In this case, depositors know that their loss in case of default will not be insured at any interest rate, and thus, equilibrium interest rate doesn't exist.

**Proposition 2:** Under appropriate parameters, there exists an interior solution for the bank's optimization problem. Equilibrium deposit demand and interest rate are:

$$D^{eq} = W \frac{2(n^H)^2 - 4a(n^H - n^L)}{4a(n^H - n^L) - (n^H)^2} \quad (3.8)$$

$$r_{eq}^d = \left( \frac{D^{eq} + W}{D^{eq}} \right) \left( \frac{(n^H)^2 - 2a(n^H - n^L)}{n^H} \right) \quad (3.9)$$

Proof: (See Appendix F)

Depositors' monitoring limits bank's leverage and profits. When the banks' leverage increases, depositors face higher default rate and bankruptcy costs. Depositors reflect these costs to deposit interest rates. Therefore, borrowing cost of each additional unit of deposit is higher than the previous one and it makes bank's profit function concave. Thus, banks' optimization problem has interior solution.

**Proposition 3:** Existence of interior solution for the bank's optimization problem depends on the upper level of return per each loan,  $n^H$ . If  $n^H < 2 \left[ a + \sqrt{a(a - n^L)} \right]$ , there will exist interior solution as given in proposition 2. Otherwise, solutions are corner solution:

$$D^{eq} = W^d \quad (3.10)$$

$$r_{eq}^d = \frac{(W^d + W)n^H - \sqrt{(W^d + W)^2 (n^H)^2 - 4aW^d(W^d + W)(n^H - n^L)}}{2W^d} \quad (3.11)$$

where  $W^d$  is the total savings of the depositors.

Proof: (See Appendix F)

Proposition 3 provides the conditions for the concavity of banks profit, and thus, interior solution to the optimization problem. If the return per loan is high enough, banks will borrow and lend all the funds in the economy which implies a corner solution for the optimization problem.

### 3.3 Asymmetric Information

In the benchmark model, there is not ex-ante heterogeneity among banks. In this section, in order to examine effects of asymmetric environment, different levels of information asymmetries are introduced into the benchmark model. Banks are differentiated into two groups with respect to their ability of generating returns from the loans. Bank's type is ex-ante known by the bank but not by depositors.

#### 3.3.1 Perfect Information and Heterogeneous Banks

Banks are classified into two groups, good and bad, according to their loan returns. Both types' returns are randomly distributed but their distribution intervals are different. Return distribution of these types are given by:

$$\text{Good banks' return: } n^G \sim \text{Uniform}[n^L + s, n^H + s]$$

$$\text{Bad banks' return: } n^B \sim \text{Uniform}[n^L - s, n^H - s]$$

where  $s$  is a positive and constant number. In perfect monitoring, depositors know banks' types, these types' return distribution and distribution intervals before depositing. But depositors still don't know return of specific bank. Therefore, solution process is similar to section 3.2.

Under perfect information, depositors ask lower interest rates from good banks for the same level of borrowing. Good banks benefit from lower rates and borrow much more than bad banks. Therefore, their default risk becomes higher than bad types and they pay higher interest rates. Compared to bad types, good banks can afford high leverages because their loan returns are adequate to pay high interest rates.

**Proposition 4:** Threshold return levels and deposit interest rates of good banks are higher than the bad banks under perfect information.

$$\tilde{n}^G > \tilde{n}^B \text{ and } (r^d)^G > (r^d)^B$$

Proof: Solution process is similar to the optimization problem defined in section 3.2. The only change is in the upper and lower level of the returns. Using the results in proposition 2 and equation (3.9), threshold returns of the banks are:

$$\tilde{n}^G = n^H + s - \frac{2a(n^H - n^L)}{n^H + s} \quad (3.12)$$

$$\tilde{n}^B = n^H - s - \frac{2a(n^H - n^L)}{n^H - s} \quad (3.13)$$



Equation (3.12) and equation (3.13) implies that threshold return of good banks is higher than the bad banks. Using equations (3.1), (3.12) and (3.13), deposit interest rates are calculated as:

$$(r^d)^G = \frac{n^H + s}{2a} \quad (3.14)$$

$$(r^d)^B = \frac{n^H - s}{2a} \quad (3.15)$$

Equation (3.14) is always higher than equation (3.15) and this shows that good banks pay higher interest rates than bad banks.

**Proposition 5:** Under perfect information good banks borrow and lend more than bad banks.

Proof: Using equations (3.8), (3.12), (3.13), (3.14) and (3.15), equilibrium deposit levels of good and bad banks are calculated as:

$$D^G = W \left( \frac{(n^H + s)^2}{4a(n^H - n^L) - (n^H + s)^2} - 1 \right) \quad (3.16)$$

$$D^B = W \left( \frac{(n^H - s)^2}{4a(n^H - n^L) - (n^H - s)^2} - 1 \right) \quad (3.17)$$

Equations (3.16) and (3.17) show that good banks borrow higher than bad banks,  $D^G > D^B$ .

### 3.3.2 Imperfect Information and Heterogeneous Banks

Perfect monitoring is an extreme assumption in the financial markets. Depositors don't have full information set about the banks; therefore, they set expectations and beliefs about the types of banks. In this section, in order to examine the asymmetric information in the financial markets, we assume depositors have imperfect monitoring power. Depositors know that there exist bad and good banks. They also know return distributions of each type. But they don't know which bank is good and which bank is bad type before depositing their money.

Good and bad banks' return distributions are same as in section 3.3.1. While depositing their money to a bank, depositors set expectations about bank's type by giving equal probability to being good and bad. There is pooling for the banks' types, and thus, good banks pay higher costs compared to perfect monitoring case. Therefore, good banks cut their optimal lending due to higher deposit interest rates. However, bad banks benefit from being in the same pool with good types. Imperfect monitoring covers up their riskiness, and hence, bad banks face with lower funding cost and raise their leverage.

**Proposition 6:** Under imperfect monitoring, bad banks have higher leverage level and pay lower interest rate compared to perfect monitoring.

$$(D^B)^I > (D^B)^P, \quad (\tilde{n}^B)^I > (\tilde{n}^B)^P, \quad (r_B^d)^I < (r_B^d)^P$$

Proof: Since depositors can't observe type of the banks, they give equal weight to being each type. Depositors' participation constraint is given by:

$$r^d \left( \frac{1}{2}(1 - F_B(\tilde{n})) + \frac{1}{2}(1 - F_G(\tilde{n})) \right) = a \quad (3.18)$$

If we rewrite equation (3.18) by using the uniform distribution assumption, it will be similar to equation (3.1). The bad bank's optimization problem can be written as:

$$\max_D \int_{\tilde{n}}^{n^H-s} (n(D+W) - r^d D) f(n) dn \quad (3.19)$$

s.t.

$$r^d(1 - F(\tilde{n})) \geq a$$

Solving the first-order condition of (3.19) yields:

$$(\tilde{n}^B)^I = \frac{n^H(n^H-s) - 2a(n^H - n^L)}{n^H - 2s} \quad (3.20)$$

If the optimization problem of bad banks is solved under perfect information, threshold return will be:

$$(\tilde{n}^B)^P = \frac{(n^H-s)(n^H-s) - 2a(n^H - n^L)}{n^H - s} \quad (3.21)$$

Comparing equations (3.20) and (3.21) implies that threshold return of bad banks is higher under imperfect information. This shows that bad type banks take more risk under imperfect monitoring. In order to compare deposit interest rates of bad banks, participation constraint is solved for the interest rates. Equation (3.18) can be rewritten as:

$$\frac{D}{D+W} (r^d)^2 - n^H r^d + a(n^H - n^L) = 0 \quad (3.22)$$

Under perfect monitoring, participation constraint of bad banks given in equation (3.22) becomes:

$$\frac{D}{D+W} (r^d)^2 - (n^H - s)r^d + a(n^H - n^L) = 0 \quad (3.23)$$

Solutions of equations (3.22) and equation (3.23) are given by:

$$(r_B^d)^I = \frac{n^H - \sqrt{(n^H)^2 - 4a(n^H - n^L) \frac{D}{D+W}}}{2 \frac{D}{D+W}} \quad (3.24)$$

$$(r_B^d)^P = \frac{(n^H - s) - \sqrt{(n^H - s)^2 - 4a(n^H - n^L) \frac{D}{D+W}}}{2 \frac{D}{D+W}} \quad (3.25)$$

Comparison of interest rates is not straightforward. When heterogeneity in banking sector disappears,  $s=0$ , interest rates under perfect and imperfect monitoring become equal. Deposit interest rates' derivative with respect to  $s$  is given by:

$$\frac{d(r_B^d)^P}{ds} = \frac{D+W}{2D} \left( -1 + \frac{(n^H - s)}{\sqrt{(n^H - s)^2 - 4a(n^H - n^L) \frac{D}{D+W}}} \right) > 0$$

$$\frac{d(r_B^d)^I}{ds} = 0$$

Signs of the derivatives imply that an increase in  $s$  raises the interest rates under perfect monitoring but it doesn't change the rates under imperfect monitoring. Therefore, difference between two interest rates is increasing with  $s$ . Thus under imperfect monitoring:

$$(r_B^d)^I < (r_B^d)^P$$

Since threshold return level is higher and deposit interest rate is lower, deposits and leverage should be higher under imperfect monitoring.

$$(D^B)^I > (D^B)^P$$

**Proposition 7:** Under imperfect monitoring, good banks' leverage is lower and they pay higher deposit interest rate compared to perfect information case.

$$(D^G)^I < (D^G)^P, \quad (\tilde{n}^G)^I < (\tilde{n}^G)^P, \quad (r_G^d)^I > (r_G^d)^P$$

Proof: Solution is same as the proof of proposition 6. The only difference is upper and lower level of loan returns. If similar steps are followed:

$$(\tilde{n}^G)^I = \frac{n^H(n^H+s)-2a(n^H-n^L)}{n^H+2s} \quad (3.26)$$

$$(\tilde{n}^G)^P = \frac{(n^H+s)(n^H+s)-2a(n^H-n^L)}{n^H+s} \quad (3.27)$$

Comparison of the equation (3.27) and (3.28) shows that  $(\tilde{n}^G)^I < (\tilde{n}^G)^P$ . Derivative of the interest rate is given by:

$$\frac{d(r_G^d)^P}{ds} = \frac{D+W}{2D} \left( 1 - \frac{n^H + s}{\sqrt{(n^H + s)^2 - 4a(n^H - n^L)} \frac{D}{D+W}} \right) < 0$$

Explanation is similar to the proof of proposition 6. Negative sign of the derivative implies that the interest rates decrease with  $s$  under perfect monitoring. However, it stays constant under imperfect monitoring. Thus  $(r_G^d)^I > (r_G^d)^P$  and  $(D^G)^I < (D^G)^P$ .

In this section, we showed that information asymmetry is important factor in lending decisions. Even small changes in the information sets of depositor and heterogeneity of banks can affect economy significantly. If weight of the bad banks is high in the economy, imperfect monitoring will cover up their riskiness, and thus, they will increase their leverage. This creates temporarily good economic environment with low rates and high lending which is similar to pre-crisis period of 2008 financial crisis. In this scenario, depositors can't insure themselves fully. This situation continues until the banks' returns from their investments are realized and exact types of the banks are observed. When the returns are realized, banks' default rate will be much higher than it should be under perfect monitoring due to excess risk taking. As a result, it will be the depositors paying the cost of such a temporary boom period and it will be the banks benefiting from imperfect monitoring.

If the weight of good banks is higher in the economy, total lending will be lower and interest rates will be higher compared to perfect monitoring. Information asymmetry causes good banks to pay extra funding costs and cut their lending. Depositors benefit from additional insurance and

banks' profits declines. This situation is similar to what is happening in US and Europe after the 2008 crisis. Governments are trying to use stress tests to prevent the information asymmetries and pooling effects. Efficiency of these solutions is examined in section 3.4.

### 3.3.3 Extreme Imperfect Information

In this section, we examine effects of extreme information asymmetries. The difference from previous sections is that depositors only know that there are two types of banks in the economy. They don't have any information regarding return distribution of these types. They estimate the return distribution of all banks as;

$$n \sim \text{Uniform}[n^L - \delta, n^H + \delta]$$

where  $\delta$  is a positive and constant noise parameter depending on depositors' beliefs on banks' returns. Therefore, when  $\delta$  is getting larger, depositors' monitoring power is decreasing and information asymmetry is increasing in the model.

Solution of the model is similar to the solution in section 3.3.1. The only difference is in the depositors' participation constraint. The new participation constraint is given by:

$$r^d(1 - F(\tilde{n})) = r^d \frac{n^H + \delta - \tilde{n}}{n^H - n^L + 2\delta} = a \quad (3.28)$$

Solution of the optimization problem with new participation constraint yields the threshold return for good and bad banks as:

$$(\tilde{n}^G)^{EI} = \frac{(n^H + \delta)(n^H + s) - 2a(n^H - n^L + 2\delta)}{n^H + 2s - \delta} \quad (3.29)$$

$$(\tilde{n}^B)^{EI} = \frac{(n^H + \delta)(n^H - s) - 2a(n^H - n^L + 2\delta)}{n^H - 2s - \delta} \quad (3.30)$$

Using equation (3.29) and (3.30), interest rates for both types of the banks are found as:

$$(r_G^d)^{EI} = (\tilde{n}^G)^{EI} \frac{D+W}{D} \quad (3.31)$$

$$(r_B^d)^{EI} = (\tilde{n}^B)^{EI} \frac{D+W}{D} \quad (3.32)$$

Notice that, noise parameter,  $\delta$  is like a blanket covering up the excess risk taking of the banks. Comparison of the extreme imperfect monitoring model with perfect monitoring model is more complex than the comparisons in the previous sections. There exist two effects, pooling of banks and noise in monitoring. These two effects might conflict for good banks depending on the banks' risk appetite. Extreme imperfect monitoring punishes good banks by putting them into same basket with bad types. This increases good banks' deposit costs. On the other hand, noise in monitoring rewards highly leveraged banks by covering up their excess risk taking behaviors. Therefore, a bank that is highly leveraged and good type might either gain or lose depending on which of these two effects is more dominant.

Noise in monitoring punishes lowly leveraged banks. If a bank chooses to be safe with low leverage, it might end up with paying higher cost compared to perfect monitoring. Depositors' beliefs for the lowest level of banks' returns could be much lower due to noise in monitoring. In



such a case, even if it is almost impossible to default, depositors estimate higher default probabilities. Following propositions show the trade-offs between these two effects and compare them for both types of banks.

**Proposition 8:** (Noise in monitoring) There exists a threshold return,  $\tilde{n}^{eq}$ , at which depositors estimate same default rate and ask same interest rate under extreme imperfect monitoring and perfect monitoring.

$$F^{EI}(\tilde{n}^{eq}) = F^P(\tilde{n}^{eq}) \text{ and } (r^d(\tilde{n}^{eq}))^{EI} = (r^d(\tilde{n}^{eq}))^P$$

If banks are highly leveraged,  $\tilde{n} > \tilde{n}^{eq}$ , depositors will estimate lower default rates for these banks under imperfect monitoring compared to perfect monitoring. If banks are lowly leveraged,  $\tilde{n} < \tilde{n}^{eq}$ , depositors will assign higher default rates under imperfect monitoring.

Proof: Equal success rate of banks under imperfect and perfect monitoring yields:

$$1 - F^{EI}(\tilde{n}^{eq}) = 1 - F^P(\tilde{n}^{eq})$$

Solution of the equation for bad and good banks yields:

$$(\tilde{n}^G)^{eq} = \frac{s(n^H - n^L)}{2\delta} + \frac{2s + n^H + n^L}{2} \quad (3.33)$$

$$(\tilde{n}^B)^{eq} = -\frac{s(n^H - n^L)}{2\delta} + \frac{-2s + n^H + n^L}{2} \quad (3.34)$$

Comparison of equation (3.33) and (3.34) implies  $(\tilde{n}^B)^{eq} < (\tilde{n}^G)^{eq}$ . Simple example can explain the importance of  $\tilde{n}^{eq}$ . Assume a good bank is highly leveraged and has maximum level of risk under perfect monitoring. Its threshold return level is given by:

$$\tilde{n} = n^H + s$$

In this scenario, since bank's default probability is 1, depositors ask infinite interest rates. However, under imperfect monitoring, depositors' belief about intervals of return distribution is wider. Therefore, depositors estimate that good bank's default probability is lower than 1. Thus, highly leveraged good bank pays lower interest rate. Solving depositor participation constraint under imperfect monitoring yields deposit interest rate as:

$$(r^d)^{EI} = a \frac{n^H - n^L + 2\delta}{\delta - s} < \infty$$

Now assume a good bank is lowly leveraged and its risk level is given by:

$$\tilde{n} = \frac{n^H + n^L + 2s}{2}$$

Deposit interest rate is  $2a$  for this risk level under perfect monitoring. In imperfect monitoring, deposit interest rate of good bank at the same risk level is given by:

$$(r^d)^{EI} = 2a \frac{n^H - n^L + 2\delta}{n^H - n^L + 2\delta - 2s} > 2a$$

Lowly leveraged bank pays higher interest cost under imperfect monitoring. Therefore, noise in monitoring induces excess risk taking among the banks.

**Proposition 9:** (Pooling effect) Pooling effect works in favor of bad banks and against good banks.

Proof: Assume banks have same risk level under perfect monitoring,  $F^P(\tilde{n}^B) = F^P(\tilde{n}^G)$ . In order to have same risk level in perfect monitoring, relation between the threshold returns should be:

$$\tilde{n}^G = \tilde{n}^B + 2s$$

In extreme imperfect monitoring, depositors estimate the success rate of banks as:

$$1 - F^{EI}(\tilde{n}^B) = \frac{n^H + \delta - \tilde{n}^B}{n^H - n^L + 2\delta} \quad (3.35)$$

$$1 - F^{EI}(\tilde{n}^G) = \frac{n^H + \delta - \tilde{n}^B - 2s}{n^H - n^L + 2\delta} \quad (3.36)$$

Success rate of good banks in equation (3.36) is lower than the success rate of bad banks in equation (3.35). Depositors couldn't differentiate type of the banks in extreme imperfect monitoring. They evaluate both banks using same return distribution. Therefore, even if both types of banks have same default risk under perfect monitoring, depositors estimate lower default risk for bad banks in extreme imperfect monitoring. Thus, depositors ask lower interest rates from bad banks compared to good banks.

**Proposition 10:** As the noise in depositors' monitoring increases, noise effect becomes more dominant than the pooling effect. When noise parameter goes to infinity, risk choices of

banks don't matter. Depositors estimate the same default rate for both types and ask the same interest rate.

$$F^{EI}(\tilde{n}^B) = F^{EI}(\tilde{n}^G) = \frac{1}{2}$$

Proof: Using the success rate formula in depositor participation constraint (3.28) and taking its limit, success rates of bad and good banks are written as:

$$\lim_{\delta \rightarrow \infty} (1 - F^{EI}(\tilde{n}^B)) = \lim_{\delta \rightarrow \infty} \left( \frac{n^H + \delta - \tilde{n}^B}{n^H - n^L + 2\delta} \right) = \frac{1}{2}$$

$$\lim_{\delta \rightarrow \infty} (1 - F^{EI}(\tilde{n}^G)) = \lim_{\delta \rightarrow \infty} \left( \frac{n^H + \delta - \tilde{n}^G}{n^H - n^L + 2\delta} \right) = \frac{1}{2}$$

When noise parameter goes to infinity, depositors lose their monitoring power on banks' returns and they estimate banks' default rate like tossing coin. Therefore, both good and bad banks, whatever their risk level is, pay the same interest rate, 2a. Interest rates are independent of leverage due to lack of monitoring. In such extreme cases, both banks want to borrow all the money in the economy and they have maximum possible leverage.

### 3.4 Government Incentives and Stress Tests

In section 3.3, we show that information asymmetries and noises in monitoring induce excess risk taking among the bad banks. Moreover, we show that these asymmetries might cause inefficiently low lending among good banks. In order to prevent these asymmetries, governments can run stress tests and announce the real types of banks. However, it is questionable that

whether governments have adequate incentives to run the tests appropriately and announce the real results. In this section, we examine situations in which it is optimal for the government to cover up stress tests' results.

**Proposition 11:** If the election time is before the realization of banks' loan returns and ratio of bad banks are high in the economy, it will be optimal for the government to maintain asymmetric information. Short term governments' optimal choice is either not to run stress tests or cover up their results.

Proof: In section 3.3, we show that bad banks are able to increase their leverage under imperfect monitoring. If ratio of bad banks is high in the economy, there will be excess lending and temporary boom period until returns of banks' investments are realized. Government can benefit from imperfect monitoring and high economic activity until elections as its popularity increases. If government announces type of banks before elections, lending will be cut and economic activity will slow down. This damages the government's popularity in the public before the elections.

Maintaining asymmetric information increases banks' default rates due to excess risk taking. Since depositors couldn't fully insure themselves in imperfect monitoring and banks have limited liability, losers are not the banks but the depositors.

### 3.4.1 Government incentives under maturity mismatch

Banks' short term borrowings and long term investments cause maturity risk in their balance sheet all the time. Such maturity risks amplify problems regarding government incentives in stress tests. In order to analyze the effects of maturity risk on government incentives, we replace standard debt contract assumption between depositors and banks with variable debt contract. We assume interest rate on deposits is variable among maturity, similar to CDS (credit default swaps). Therefore, depositors can update interest rate depending on new information sets. On the other hand, banks' hands are tied as they can't change their borrowing amount. As a result, banks need to finance their short term debt each period with new interest rates. Thus, they are exposed to interest rate risks.

In this section, we examine a model similar to the one presented in section 3.3.1 in three periods,  $t = 0, 1, 2$ . In period 0, banks borrow deposits,  $D$  with the interest rate,  $r_0^d$  to finance their lending. Banks can't change their deposit level until period 2. In period 1, depositors update deposit rates,  $r_1^d$  with respect to their new information set. They recalculate their participation constraints. In period 2, banks' loan returns are realized and they pay their debt. In this set up, question is whether it is optimal for government to run stress test and announce the results in period 1.

**Proposition 12:** If the ratio of bad banks is high in the financial sector with maturity risk, there will be no incentive for a short term government to announce type of banks.

Proof: If depositors learn the type of the banks in period 1, they will ask higher interest rates from bad banks which have already taken excess risk under imperfect economy at period 0. Since banks can't re-optimize their deposit levels in period 1, their profits significantly decline and default rates rise to extremely high levels at the end of period 2. Banks pay the cost of excess risk taking. It also damages the economy since lending drops and government might need to bail out default institutions. Short term governments don't choose this option. Instead they prefer not to run stress test or to run biased stress tests. In this case, default rates are lower in period 2. However, depositors are not able to insure themselves and they pay the cost of excess risk taking. Banks benefit from high leverages. Government prevents the high default rates and bail outs. On the other hand, government loses its reliability and problems with asymmetric information mentioned in previous sections continue in the long term. But, since government is selected for short term, it ignores such long term damages. Open form of participation constraint in period 0 is given by:

$$\frac{D}{D+W}(r_0^d)^2 - n^H r_0^d + a(n^H - n^L) = 0 \quad (3.37)$$

In period 1, if depositors learn that the banks are bad type, they will update the participation constraint as:

$$\frac{D}{D+W}(r_1^d)^2 - (n^H - s)r_1^d + a(n^H - n^L) = 0 \quad (3.38)$$

Participation constraints (3.37) and (3.38) yield the interest rate at period 0 and 1:

$$r_0^d = \frac{n^H - \sqrt{(n^H)^2 - 4a(n^H - n^L) \frac{D}{D+W}}}{2 \frac{D}{D+W}} \quad (3.39)$$

$$r_1^d = \frac{n^H - s - \sqrt{(n^H - s)^2 - 4a(n^H - n^L) \frac{D}{D+W}}}{2 \frac{D}{D+W}} \quad (3.40)$$

Notice that interest rates in equation (3.39) and (3.40) are equal when banks are homogeneous,  $s=0$ . Deposit interest rate in period 1 increases with  $s$ ,  $\frac{dr_1^d}{ds} > 0$ . It implies that interest rate in period 1 is higher than the rate in period 0,  $r_1^d > r_0^d$ , and difference is increasing with  $s$ . Higher  $s$  means lower return of bad banks. Therefore, bad banks' extra interest rate cost in period 1 increases with  $s$ . As a result, such an increase in costs cause higher default rates at the end of period 2.

### 3.5 Conclusion

Banks are originally designed to decrease inefficiency and risks in the economy. However, during the 2008 crisis, they were the source of problems. They went around the regulations by using the innovations such as derivatives and they took excess risk. Depositors were not aware of the situation due to asymmetric information and noises in monitoring. This paper presents an analytical framework to analyze this situation. We examine the behavior of banks under the



asymmetric information with various levels imperfect monitoring. We use the single period model with bankruptcy costs to evaluate banks' optimal decisions.

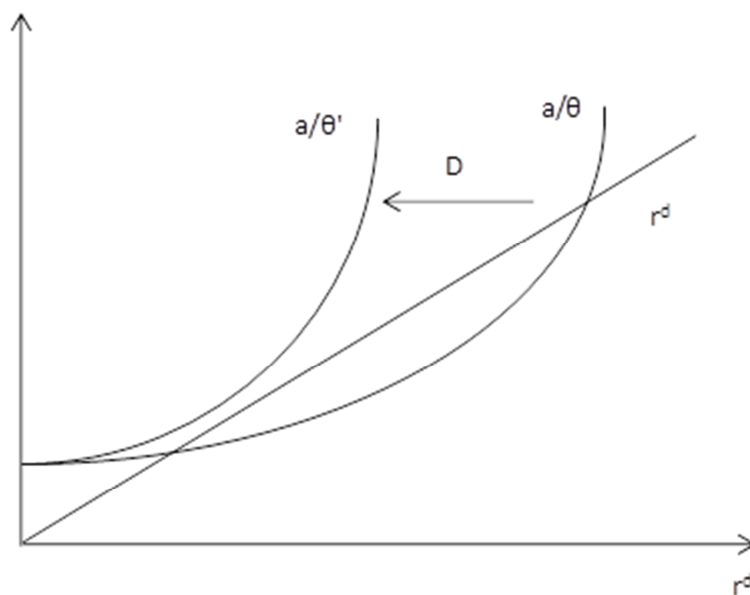
We solve our model analytically under both perfect monitoring and imperfect monitoring with heterogeneous banks. We find that there are two effects shaping the decisions of banks, pooling effect and noise in monitoring. When depositors don't know the type of the banks, they assign same probabilities to being in any type and this causes pooling effect. We show that pooling effect induces excess risk taking among bad banks and curbs good banks' lending. If depositors don't have exact information about banks' return distributions, they will set up beliefs like adding a noise to their monitoring power. We show that noise in monitoring causes depositors to overestimate default risks of lowly leveraged banks and underestimate default risks of highly leveraged banks. Therefore, highly leveraged banks' borrowing cost is lower compared to perfect monitoring case. As a result, noise in monitoring induces excess risk taking among the banks.

Pooling and noise effects work in same direction for bad banks and encourage them to raise their risk level. However, their effects on good banks conflict. These results explain analytically banks' behaviors before 2008 financial crisis. Before the 2008 crisis, due to lack of perfect monitoring over banks performance, especially bad type banks took excess risk.

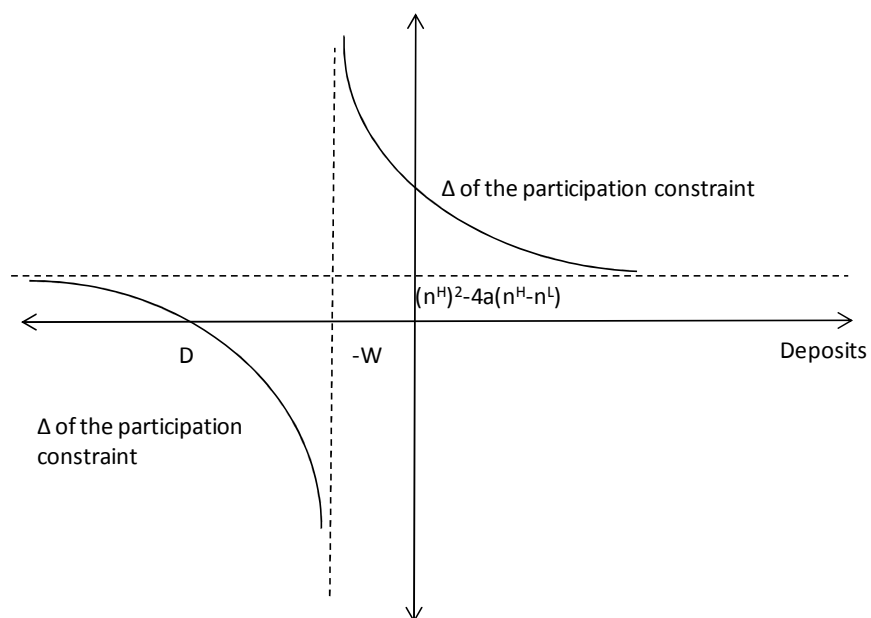
Although banks decreased their leverage after the 2008 crisis, depositors lost confidence to the banks. Governments used stress tests to prevent asymmetric information and to gain depositors' confidence again. In the last section, we analytically show that if government is

elected for short term and the bad banks is widespread in the economy, government doesn't have any incentive to announce real type of the banks. Banks' maturity mismatching problems banks amplify this result. If the government is forced to run such tests by public pressure, it will have incentives to present biased results. Such a situation decreases reliability of stress tests in the future.

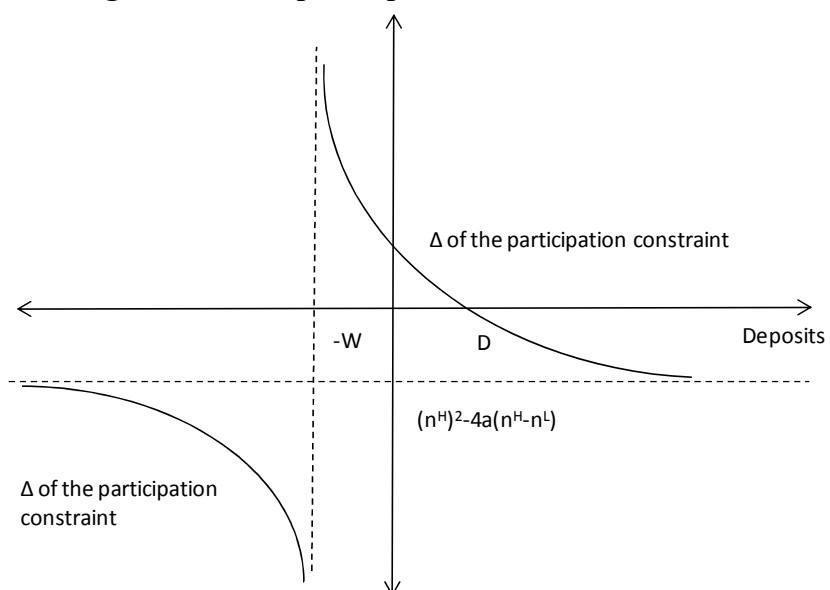
**Figure 3.1: Existence of solution to participation constraint**



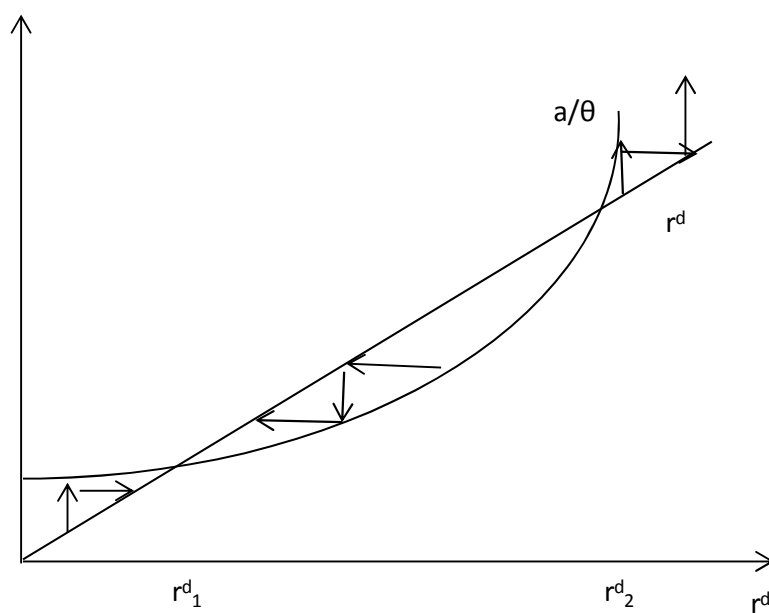
**Figure 3.2:  $\Delta$  of participation constraint when  $D < 0$**



**Figure 3.3:  $\Delta$  of participation constraint when  $D > 0$**



**Figure 3.4: Solution of the depositors' participation constraint**



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# Appendix A: Solution of Decentralized Model

## A.1 Financial Contract

Depositor's participation constraint in financial constraint is always binding and  $a_t$  clears the market and equalizes total saving of depositors to total deposit demand from the banks,  $(1-\eta)d_t^s = \eta d_t$ . Given  $q_{t+1}$ , deposit interest rate ( $r_{t+1}^d$ ) is determined with this equation:

$$q_{t+1} l_{t+1} \pi_{t+1}^D = a_{t+1} d_{t+1}$$

Banks distribute the revenues from capital good sales. Their budget constraint:

$$q_{t+1} l_{t+1} \bar{n} - q_{t+1} l_{t+1} \pi_{t+1}^D - \mu q_{t+1} l_{t+1} \int_{n_L}^{\tilde{n}_{t+1}} n_{t+1} f(n) dn = q_{t+1} l_{t+1} \pi_{t+1}^B$$

Financial contract is solved by banks by embedding depositor's participation constraint and resource constraint into their objective function. Given  $q_{t+1}$ , objective function becomes:

$$\max_{l_{t+1}} q_{t+1} l_{t+1} \bar{n} - \mu q_{t+1} l_{t+1} \int_{n_L}^{\tilde{n}_{t+1}} n_{t+1} f(n) dn - a_{t+1} d_{t+1}$$

Assuming uniform distribution of  $f(n)$  implies the first order condition:

$$a_{t+1} - q_{t+1} \left[ \bar{n} - \mu F(\tilde{n}_{t+1}) \frac{\tilde{n}_{t+1} + n_L}{2} - \mu l_{t+1} F(\tilde{n}_{t+1}) \frac{\tilde{n}_{t+1}}{n_H - n_L} \frac{d\tilde{n}_{t+1}}{dl_{t+1}} \right] = 0$$

Direct derivative of objective function implies:

$$-\frac{1}{l_{t+1}} \frac{\pi_{t+1}^B}{(\pi_{t+1}^B)'} = \frac{d\tilde{n}_{t+1}}{dl_{t+1}}$$

Rewriting the first order condition:

$$\text{FOC } (l_{t+1}): \quad a_{t+1} - q_{t+1} \left[ \bar{n} - \mu \frac{(\tilde{n}_{t+1})^2 - (n_L)^2}{2(n_H - n_L)} + \mu \frac{\tilde{n}_{t+1}}{n_H - n_L} \frac{\pi_{t+1}^B}{(\pi_{t+1}^B)'} \right] = 0$$

The first order condition implies that  $\tilde{n}$  is the function of  $q$  and  $a$ . It doesn't depend on bank capital,  $w$ . Also depositor's participation constraint gives  $r^d$  as function of  $\tilde{n}$ . Therefore, all atomistic banks, regardless of their capital choose the same leverage and risk level,  $\tilde{n}$ .

In order to ensure that the solution is the maximum point in the domain, second order condition is examined. Second order condition is negative if:

$$\frac{d\tilde{n}_{t+1}}{dl_{t+1}} + 2\left(\frac{d\tilde{n}_{t+1}}{dl_{t+1}}\right)^2 l_{t+1} + \tilde{n}_{t+1} l_{t+1} \frac{d^2 \tilde{n}_{t+1}}{dl_{t+1}^2} > 0$$

Assuming  $\frac{d^2 r_{t+1}^d}{dl_{t+1}^2}$  is small enough, the given condition holds and solution of the first order

condition is the maximum point.

## A.2 Shareholders' Problem

Shareholders' problem can be rewritten by embedding their budget constraint into their objective function. The Lagrangian becomes:

$$L_t = E_t \sum_{s=t}^{\infty} (\beta\gamma)^{t+s} ((q_s \pi_s^B l_s \frac{1_s}{w_s}) w_s - w_{s+1} + w_s^B)$$

Note that the leverage,  $l_s/w_s$ , is independent of initial capital  $w_s$ . In the financial contract, we show that all atomistic banks choose the same leverage and risk level,  $\tilde{n}$ , regardless of their initial capital. Thus, internal return rate,  $\rho_t$  is not a function of  $w_s$ :

$$\rho_s = q_s \pi_s^B l_s \frac{1_s}{w_s}$$

This implies the first order condition:

$$\text{FOC } (w_{t+1}): \quad \frac{1}{\beta\gamma} = q_{t+1} \pi_{t+1}^B \frac{l_{t+1}}{w_{t+1}}$$

### A.3 Depositors' Problem

We assume depositors invest in all banks to insure themselves against idiosyncratic uncertainty among banks. The average return of depositor from bank deposits is calculated by averaging banks' deposit repayments:

$$q_t l_t \pi_t^D \frac{\eta}{1-\eta} = \sum_j \left\{ (1-F(\tilde{n}_{j,t})) r_{j,t}^d d_{i,j,t}^s + (1-\mu) q_t \frac{l_{j,t}}{d_{j,t}} d_{i,j,t}^s \int_{n_L}^{\tilde{n}_{j,t}} n_t f(n) dn \right\}$$

where index  $j$  represents each atomistic bank, and index  $i$  represents each depositor.  $d_{i,j,t}^s$  is the deposits of depositor  $i$  in bank  $j$  and  $d_{j,t}$  is the total deposits in bank  $j$ . In the financial

contract, we show that banks are similar in terms of risk and leverage. Since the depositors are homogeneous, aggregation simplifies average depositors' return equation:

$$q_t l_t \pi_t^D \frac{\eta}{1-\eta} = (1-F(\tilde{n}_t)) r_t^d d_t^s + (1-\mu) q_t \frac{L_t}{D_t} d_t^s \int_{n_L}^{\tilde{n}_t} n_t f(n) dn$$

where  $L_t$  is the total lending and  $D_t$  is the total deposits in the economy. Since all banks have same leverage, average bank's leverage is equal to the leverage level of overall banking industry.

Depositor maximizes its utility, equation (1.5), subject to budget constraint, equation (1.6). Solving optimization problem using the average depositors' return equation yields:

$$\text{FOC } (d_{t+1}^s): \quad u_{c,t} = \beta E_t u_{c,t+1} \left( (1-F(\tilde{n}_{t+1})) r_{t+1}^d + (1-\mu) q_{t+1} \frac{l_{t+1}}{d_{t+1}} \int_{n_L}^{\tilde{n}_{t+1}} n_{t+1} f(n) dn \right)$$

#### A.4 Model Optimality Conditions

$$u_{c,t} = \beta E_t u_{c,t+1} \left( (1-F(\tilde{n}_{t+1})) r_{t+1}^d + (1-\mu) q_{t+1} \frac{l_{t+1}}{d_{t+1}} \int_{n_L}^{\tilde{n}_{t+1}} n_{t+1} f(n) dn \right) \quad (\text{A.1})$$

$$E_t \left[ q_{t+1} (\bar{n} - \mu \frac{(\tilde{n}_{t+1})^2 - (n^L)^2}{2(n^H - n^L)} + \mu \frac{\tilde{n}_{t+1}}{n^H - n^L} \frac{\pi_{t+1}^B}{\pi_{t+1}^B}) - a_{t+1} \right] = 0 \quad (\text{A.2})$$

$$\frac{1}{\beta \gamma} = E_t q_{t+1} \pi_{t+1}^B \frac{l_{t+1}}{w_{t+1}} \quad (\text{A.3})$$

$$E_t \left[ q_{t+1} l_{t+1} \pi_{t+1}^D - a_{t+1} d_{t+1} \right] = 0 \quad (\text{A.4})$$

$$d_{t+1}^s + c_t = w_t^d h_t + q_t l_t \pi_t^D \eta / (1 - \eta) \quad (\text{A.5})$$

$$w_{t+1} + c_t^B = q_t l_t \pi_t^B + w_t^B \quad (\text{A.6})$$

$$w_t^d = (1 - \alpha - \alpha_H) \frac{Y_t}{H_t} \quad (\text{A.7})$$

$$w_t^B = \alpha_H \frac{Y_t}{\eta_t} \quad (\text{A.8})$$

$$q_t = \alpha \theta_t (K_t)^{\alpha-1} (H_t)^{1-\alpha-\alpha_H} (\eta)^{\alpha_H} \quad (\text{A.9})$$

$$H_t = (1 - \eta) h_t \quad (\text{A.10})$$

$$K_t = \eta \bar{n} l_t \quad (\text{A.11})$$

$$Y_t = (1 - \eta) c_t^D + \eta c_t^B + \eta F(\tilde{n}_t) \mu q_t l_t + \eta l_{t+1} \quad (\text{A.12})$$

$$(1 - \eta) d_t^s = \eta d_t \quad (\text{A.13})$$

$$\log \theta_{t+1} = \rho \log \theta_t + \varepsilon_t \quad (\text{A.14})$$

$$\pi_{t+1}^B = \int_{\tilde{n}_{t+1}}^{n_H} n_{t+1} f(n) dn - (1 - F(\tilde{n}_{t+1})) \tilde{n}_{t+1} \quad (\text{A.15})$$

$$\pi_{t+1}^D = \int_{n_L}^{\tilde{n}_{t+1}} (1 - \mu) n_{t+1} f(n) dn + (1 - F(\tilde{n}_{t+1})) \tilde{n}_{t+1} \quad (\text{A.16})$$

$$\tilde{n}_t = \frac{r_t^d d_t}{q_t (d_t + w_t)} \quad (\text{A.17})$$

## A.5 Steady State Equations

$$\theta=1 \text{ and } a=1/\beta \quad (\text{A.18})$$

$$q - \frac{a}{\bar{n} - \mu \int_{n_L}^{\tilde{n}} n f(n) dn + \mu \tilde{n} f(\tilde{n}) \frac{\pi^B}{\pi^{B'}}} = 0 \quad (\text{A.19})$$

(solves for  $\tilde{n}(q)$ )

$$\frac{\gamma q \pi^B a}{a - q \pi^D} = \frac{1}{\beta} \quad (\text{A.20})$$

(solves for  $q$ )

$$K = \left( \frac{q}{\alpha \theta H^{1-\alpha-\alpha_H} \eta^{\alpha_H}} \right)^{\frac{1}{\alpha-1}} \quad (\text{A.21})$$

$$l = \frac{K}{\eta \bar{n}} \quad (\text{A.22})$$

$$Y = \theta K^\alpha H^{1-\alpha-\alpha_H} \eta^{\alpha_H} \quad (\text{A.23})$$

$$w = (a - q \pi^D) \frac{1}{a} \quad (\text{A.24})$$

$$d = l - w \quad (\text{A.25})$$

$$c^D = (a-1)d \frac{\eta}{1-\eta} + (1-\alpha-\alpha_H) \frac{Y}{1-\eta} \quad (\text{A.26})$$

$$r^d = \frac{\tilde{n} q l}{d} \quad (\text{A.27})$$

$$H=(1-\eta)$$

(A.28)

$$c^B = q\pi^B l + \frac{\alpha_H Y}{\eta} - w$$

(A.29)

## Appendix B: Existence and Uniqueness of Decentralized Equilibrium

In Appendix B, we prove existence of solution in the decentralized model. We also determine conditions for the uniqueness of the equilibrium. In order to prove existence of equilibrium, showing existence of fixed point (steady state) where economy converges is adequate. According to the steady state equations in Appendix A.5, all other variables of the model are one-to-one function of asset price,  $q$ , and threshold risk level,  $\tilde{n}$ . Therefore, proving the existence and uniqueness of threshold risk level and asset price implies the existence and uniqueness of equilibrium.

Consider the steady state optimality equations for financial contract in Appendix A.5:

$$q \left[ \bar{n} - \mu \int_{n^L}^{\tilde{n}} n f(n) dn + \mu \tilde{n} f(\tilde{n}) \frac{\pi^B}{\pi^{B'}} \right] = a$$

Assuming uniform distribution of  $n$  implies:

$$\pi^B = \left( \int_{\tilde{n}}^{n^H} n f(n) dn - (1 - F(\tilde{n})) \tilde{n} \right) = \frac{(n^H - \tilde{n})^2}{2(n^H - n^L)}$$

$$\frac{d\pi^B}{d\tilde{n}} = -\frac{n^H - \tilde{n}}{n^H - n^L}$$



Embedding these equations into the optimality condition of financial contract above yields:

$$q = \frac{a}{\bar{n} + \frac{\mu(n^L)^2 - \mu n^H \bar{n}}{2(n^H - n^L)}}$$

In this equation, we write asset prices as one-to-one function of threshold risk level,  $\tilde{n}$ . The next step of proving the existence of equilibrium is the derivation of  $\tilde{n}$ . The shareholder's optimality condition at steady state is given by:

$$\frac{1}{\beta\gamma} = \frac{q\pi^B a}{a - q\pi^D}$$

Assumption of uniform distribution implies:

$$\pi^D = \int_{n^L}^{\tilde{n}} (1-\mu)nf(n)dn + (1-F(\tilde{n}))\tilde{n} = \frac{(1-\mu)(\tilde{n}^2 - (n^L)^2) + 2(n^H - \tilde{n})\tilde{n}}{2(n^H - n^L)}$$

Embedding this equation into shareholder's inter-temporal equation yields:

$$\frac{1}{\beta\gamma} = \frac{qa(n^H - \tilde{n})^2}{2a(n^H - n^L) + q(1+\mu)\tilde{n}^2 + q(1-\mu)(n^L)^2 - 2qn^H\tilde{n}}$$

If the  $q$  is replaced with its functional form, second order polynomial equation of threshold risk level is obtained:

$$(1+\mu-\gamma)\tilde{n}^2 - 2(n^H(1+\mu/2-\gamma))\tilde{n} + 2(n^H - n^L)\bar{n} + (n^L)^2 - \gamma(n^H)^2 = 0$$

Solution of this second order equation exists and there are two solutions (assuming  $n^L=0$  for simplification):

$$\tilde{n}_1 = \frac{n^H}{1+\mu-\gamma}(1-\gamma) \text{ and } \tilde{n}_2 = n^H$$

Second root of the equation is eliminated, since bank's profit is zero at this root and also shareholder's inter-temporal equation is not well defined and not satisfied. Therefore, the only solution for threshold level is:

$$\tilde{n}_1 = \frac{n^H}{1+\mu-\gamma}(1-\gamma)$$

As a result, we prove that solution for threshold risk level exists and it is unique. As asset price function and other variables are one-to one function of threshold risk level, it implies that equilibrium for the decentralized model exists and it is unique.

## Appendix C: Solution of Planner's Problem with Full Capability

The complexity with planner's problem is the choice of objective function, as there are two agents; shareholders and depositors. In order to simplify the problem, we assume planner's goal is to maximize depositor's utility while keeping shareholder's lifetime utility same as the decentralized equilibrium level. Utility constraint of shareholders:

$$U^B \geq (U^B)^{DE}$$

This constraint is always binding in the problem of planner with full capability. Constraint is rewritten as:

$$c^B = \bar{c}^B \text{ when } U^B = (U^B)^{DE}$$

where  $\bar{c}^B$  is the constant consumption level of shareholders to ensure their lifetime utility level in decentralized equilibrium. Replacing the utility constraint, planner solves the Lagrangian:

$$L_t = E_t \sum_{s=t}^{\infty} (\beta)^{t+s} \left[ u(c_s^D) + \lambda_s (\theta_s (\bar{n}\eta l_s)^\alpha H^{1-\alpha-\alpha_H} \eta^{\alpha_H} - (1-\eta)c_s^D - \eta\bar{c}_s^B - \eta l_{s+1}) \right]$$

This implies the first order condition as:

$$\text{FOC : } \eta = \beta E_t \left[ (c_{t+1}^D / c_t^D)^{-\sigma} \alpha \theta_{t+1} (\bar{n}\eta)^{\alpha} H^{1-\alpha-\alpha_H} \eta^{\alpha_H} l_{t+1}^{\alpha-1} \right]$$

Budget constraint, production technology and first order conditions constitute planner optimality conditions and steady state equations.

*Planner's (Full Capability) Optimality Conditions:*

$$\eta = \beta E_t \left[ (c_{t+1}^D / c_t^D)^{-\sigma} \alpha \theta_{t+1} (\bar{n}\eta)^{\alpha} H^{1-\alpha-\alpha_H} \eta^{\alpha_H} l_{t+1}^{\alpha-1} \right] \quad (\text{C.1})$$

$$\theta_t (\bar{n}\eta)^{\alpha} H^{1-\alpha-\alpha_H} \eta^{\alpha_H} l_t^{\alpha} - (1-\eta) c_t^D - \eta \bar{c}_t^B - \eta l_{t+1} = 0 \quad (\text{C.2})$$

$$\log \theta_{t+1} = \rho \log \theta_t + \varepsilon_t \quad (\text{C.3})$$

$$K_t = \eta \bar{n} l_t \quad (\text{C.4})$$

$$Y_t = \theta_t (K_t)^{\alpha} (H_t)^{1-\alpha-\alpha_H} (\eta)^{\alpha_H} \quad (\text{C.5})$$

$$q_t = \alpha \theta_t (K_t)^{\alpha-1} (H_t)^{1-\alpha-\alpha_H} (\eta)^{\alpha_H} \quad (\text{C.6})$$

*Planner's (Full Capability) Steady State Equations:*

$$\theta=1 \text{ and } a=1/\beta \quad (\text{C.7})$$

$$l^{\alpha-1} = \frac{1}{\beta \alpha \theta \bar{n}^{\alpha} H^{1-\alpha-\alpha_H} \eta^{\alpha+\alpha_H-1}} \quad (\text{C.8})$$

$$c^D = \frac{\theta (\bar{n}\eta)^{\alpha} H^{1-\alpha-\alpha_H} \eta^{\alpha_H} l^{\alpha} - \eta \bar{c}^B - \eta l}{1-\eta} \quad (\text{C.9})$$

$$Y = \theta K^{\alpha} H^{1-\alpha-\alpha_H} \eta^{\alpha_H} \quad (\text{C.10})$$

$$l = \frac{K}{\eta \bar{n}} \quad (\text{C.11})$$

$$q = \alpha \theta K^{\alpha-1} H^{1-\alpha-\alpha_H} \eta^{\alpha_H} \quad (\text{C.12})$$

Since these functions have unique solutions in the given domain, solution to the planner's problem exists and it is unique.

# Appendix D: Solution of Planner's Problem with Restricted Capability

Social planner with restricted capability doesn't have any control on depositor's problem and it is constrained with the financial contract. Therefore, social planner's goal is to maximize banks' profit while keeping utility of depositors at least indifferent.

Considering the optimization problem of restricted planner, if the utility constraint of depositors ( $U^D \geq (U^D)^{DE}$ ) is not binding, solution will be interior solution otherwise it will be corner solution.

## D.1 Existence of Equilibrium in Interior Solutions

Solution of restricted planner's problem while depositor's utility is not binding is similar to the solution of decentralized economy. Optimality conditions and steady state equations are same as the equations in Appendix A.4 and A.5. The only difference is the first order condition of financial contract. We can rewrite it for restricted planner as:

$$\alpha q_{t+1}^S (\bar{n} - \mu \frac{(\tilde{n}_{t+1})^2 - (n^L)^2}{2(n^H - n^L)} + \mu \frac{\tilde{n}_{t+1}}{n^H - n^L} \frac{\pi_{t+1}^B}{\pi_{t+1}^B}) = a_{t+1}$$

and the steady state version of the equation is:

$$\alpha q^s (\bar{n} - \mu \frac{(\tilde{n})^2 - (n^L)^2}{2(n^H - n^L)} + \mu \frac{\tilde{n}}{n^H - n^L} \frac{\pi^B}{\pi^{B'}}) = a$$

All other equations are similar to the equations in decentralized solution. The way of proving existence of equilibrium is also similar. Uniqueness of threshold risk level and asset price implies the existence and uniqueness of equilibrium. Considering the new optimality condition related with the financial contract, asset price equation is rewritten as:

$$q = \frac{a}{\alpha \left( \bar{n} + \frac{\mu(n^L)^2 - \mu n^H \tilde{n}}{2(n^H - n^L)} \right)}$$

Using the new form of the asset price function, the new equation for threshold of risk level is obtained as:

$$(1 + \mu - \gamma)\tilde{n}^2 - 2(1 + \mu\alpha/2 - \gamma)n^H\tilde{n} + 2\alpha(n^H - n^L)\bar{n} + (1 - \mu + \alpha\mu)(n^L)^2 - \gamma(n^H)^2 = 0$$

Solution of the second order equation exists and there are two solutions (assuming  $n^L=0$  for simplification):

$$\tilde{n}_{1,2} = \frac{n^H}{1 + \mu - \gamma} \left[ 1 + \mu\alpha/2 - \gamma \pm \sqrt{(1 - \alpha)(1 + \mu\gamma - \gamma) + \mu^2\alpha^2/4} \right]$$

Since  $\alpha$  and  $\mu$  are defined between 0 and 1, inside of square root is always positive and solution for threshold risk level always exists. Uniqueness of the equilibrium depends on the parameter values, since there exist two solutions for  $\tilde{n}$ . For the chosen model parameters in this

paper, one of the solutions for  $\tilde{n}$  is always greater than  $n^H$  and this solution is not feasible. Therefore, solution is unique.

## D.2 Existence of Equilibrium in Corner Solutions

Considering the binding utility constraint of depositors, there exists a consumption level which keeps depositors indifferent, such as

$$c^D = \bar{c}^D \text{ where } U = U^{DE}$$

If the utility constraint is binding, it implies that planner's optimal choice of leverage is too low for depositors. Low lending means high prices and lower wages in the future. Therefore, social planner should deviate from its optimum decision by increasing the leverage. There exists a parameter,  $\hat{\alpha}$ , for each  $\bar{c}^D$  which forces planner to choose a higher leverage level and converts planner's solution into an interior solution. Notice that  $\alpha$  is replaced with  $\hat{\alpha}$  only in first order condition of financial contract. The new equation for interior solution can be rewritten as:

$$q = \frac{a}{\hat{\alpha} \left( \bar{n} + \frac{\mu(n^L)^2 - \mu n^H \tilde{n}}{2(n^H - n^L)} \right)}$$

The new form of optimization problem doesn't have binding constraints. However, its solution is still in the feasible set of the previous form of problem, optimization problem with binding constraint. Such a heuristic approach allows us to estimate corner solutions of problems



with binding constraints. The rest of the proof is similar to the one in the interior solution section.

### D.3 Proof of Propositions

#### *Proof of Proposition 1*

Second order condition of the financial contract in restricted planner's problem is negative. Its proof is similar to the proof in Appendix A.1 The negativity of the second order condition implies that social planner's solution is the optimum for the bank profits. Bank profits in restricted planner's solution are higher than the profits in decentralized solution, and thus, shareholders are pareto better in restricted planner's solution. Restricted planner improves bank profits by lowering leverage and default rates, and so, increasing the steady state levels of asset prices.

While solving the model, second order approximation methodology is chosen to take into account volatility of variables in utilities. Second order approximations of depositor's expected utility around the steady state in social planner (SP) and decentralized equilibrium (DE) are given by:

$$E(u(c^{SP})) \approx u(\bar{c}^{SP}) + u'(\bar{c}^{SP})E(c - \bar{c}^{SP}) + 1/2u''(\bar{c}^{SP})(\sigma_c^2)^{SP}$$

$$E(u(c^{DE})) \approx u(\bar{c}^{DE}) + u'(\bar{c}^{DE})E(c - \bar{c}^{DE}) + 1/2u''(\bar{c}^{DE})(\sigma_c^2)^{DE}$$

where  $\bar{c}$  is the consumption at the steady state and  $\sigma_c^2$  is the variance of consumption around the steady state levels. The source of volatility in the consumption is the variance of shocks and there is direct mapping ( $\tilde{\sigma}_c^2 \rightarrow \tilde{\sigma}^2$ ). If there is no aggregate uncertainty in the model, utility of agents will be equal to the steady state utility level,  $u(\bar{c})$ . Since there is stochastic environment and depositors are risk averse, second and third terms in the utility equations will not be equal to zero.  $u'(\bar{c})E(c-\bar{c}) + 1/2u''(\bar{c})\sigma_c^2$  is the effect of the stochastic consumption on agents' utility functions.

Restricted planner's solution has two different effects on depositors' utility. The first effect is on steady state levels of depositor's consumption. Low supply of capital goods decreases wages and it causes a direct decrease in steady state values of depositors' consumption in planner's problem ( $u(\bar{c}^{DE}) - u(\bar{c}^{SP}) > 0$ ). The second effect is regarding the stochastic consumption. In stochastic environment, low default rates of banks allow them to accumulate more capital in the long run compared to competitive equilibrium. Therefore, even with low leverage, banks can lend more in planner's problem. Moreover, banks have more robust structure in planner's problem. Therefore, volatility in depositors' deposit returns is lower. As a result, negative contribution of stochastic environment in social planner's problem is limited compared to its effects in decentralized equilibrium:

$$u'(\bar{c}^{SP})E(c-\bar{c}^{SP}) + \frac{1}{2}u''(\bar{c}^{SP})(\sigma_c^2)^{SP} > u'(\bar{c}^{DE})E(c-\bar{c}^{DE}) + \frac{1}{2}u''(\bar{c}^{DE})(\sigma_c^2)^{DE}$$

$$\text{Given } A = u'(\bar{c})E(c-\bar{c}) + \frac{1}{2}u''(\bar{c})(\sigma_c^2) :$$

$$\frac{dA^{SP}}{d\sigma} > \frac{dA^{DE}}{d\sigma}$$

These results are supported with the simulation results given in Figure 1.10, Figure 1.11 and Figure 1.12.

There exist a threshold level of variance of shocks,  $\tilde{\sigma}^2$ , which equates utility loss at steady state (left side of equation) to gains from choosing low risk levels (right side of the equation):

$$u(\bar{c}^{DE}) - u(\bar{c}^{SP}) = u'(\bar{c}^{SP})E(c-\bar{c}^{SP}) - u'(\bar{c}^{DE})E(c-\bar{c}^{DE}) + \frac{1}{2}[u''(\bar{c}^{SP})(\sigma_c^2)^{SP} - u''(\bar{c}^{DE})(\sigma_c^2)^{DE}]$$

If variance of shock is higher than this threshold level, the right side of the equation will be higher than the left side. Therefore, social planner's solution will be better off for depositors and there will be interior solution to the planner's problem. As bankers are also pareto better, decentralized equilibrium is not constrained efficient. Simulations in Figure 1.10 show that threshold level of variance of shocks is 0.35 under benchmark calibration. Simulations in Figure 1.11 also supports that both agents will be pareto better in planner's problem if variance of shocks are greater than this threshold level.

### *Proof of Proposition 2*

Proposition 1 implies that if the variance of shocks is higher than the threshold level, depositors' constraint in planner's solution will not be binding and both agents will be pareto

better. Proposition 2 generalizes the inefficiency of decentralized equilibrium and claims that even if the variance of shock is lower than the threshold level, planner can always choose pareto better allocations.

The first step in proof of proposition 2 is to show that regardless of variance of shocks, a marginal cut in lending make depositors better off in decentralized equilibrium. Such a marginal decrease in lending causes infinite small decline in steady state consumption,  $\bar{c}^{DE}$ :

$$\bar{c}^{DE} \approx \bar{c}^{DE} - d\bar{c}^{DE} \quad \text{and} \quad \Delta u(\bar{c}^{DE}) \approx 0$$

But the contribution of aggregate shocks in depositors' utility,  $A^{DE}$ , is more sensitive to such a decline in lending as it leads to more robust banking sector. Combining these results yield:

$$\Delta A^{DE} > 0, \Delta l < 0 \quad \text{and} \quad \Delta A^{DE} > \Delta u(\bar{c}^{DE})$$

Considering the second order approximation of depositors' utility in the proof of proposition 1, these results imply:

$$\frac{dU^{DE}}{dl} < 0$$

As a result, after a marginal cut in lending, depositors are better off. The second step is to show that bankers' are also pareto better. Appendix D.2 shows that we can generate a marginal cut in banks' lending by transforming optimization problem. If choose an  $\alpha$  slightly smaller than 1, we can marginally cut banks' lending in decentralized equilibrium. In addition, optimization problem becomes in the same format as restricted planner's problem with interior solution.

Appendix D.1 proves the existence of maximum point for such problems. Therefore, this shows that shareholders' can increase their utility by cutting their lending in decentralized equilibrium marginally. Simulations with benchmark parameters in Figure 1.11 and Figure 1.12 support these results.

# Appendix E: Depositor Participation

## Constraint

### E.1 Existence of Solution

Depositors' participation condition can be rewritten as:

$$\frac{a}{\theta} = r^d \quad (E.1)$$

where  $\theta$  is the success rate of the banks and  $\theta = \frac{1}{n^H - n^L} (n^H - \frac{r^d D}{D+W})$ . Figure 3.1 displays the importance of  $\theta$  for the existence of solution to the participation constraint. If the  $\theta$  is smaller than a threshold level, equilibrium interest rate will not exist. It implies that depositors don't accept to deposit their money, since banks' default probability is too high. Open form of equation (E.1) is written as:

$$\frac{D}{D+W} (r^d)^2 - n^H r^d + a(n^H - n^L) = 0 \quad (E.2)$$

Equation (E.2) is a second order polynomial equation. Condition to have a real solution is given by:

$$(n^H)^2 - 4 \frac{D}{D+W} a(n^H - n^L) \geq 0 \quad (E.3)$$

Equation (E.3) is binding at the threshold level of deposits,  $\hat{D}$  that is critical to determine the solutions:

$$\hat{D} = \frac{(n^H)^2 W}{4a(n^H - n^L) - (n^H)^2}$$

If upper level of returns are higher than a critical level,  $n^H > \hat{n}$  where  $\hat{n} = 2 \left[ a + \sqrt{a(a - n^L)} \right]$ , threshold level of deposits will be always negative,  $\hat{D} < 0$ . Figure 3.2 displays the graphical explanation of this condition. Notice that hyperbole denotes  $\Delta$  of equation (E.2) and it intersects x axis when deposits are at the threshold level,  $\hat{D}$ . In Figure 3.2, it is clear that  $\hat{D} < 0$  and upper side of hyperbole never intersects x axis. Thus,  $\Delta$  is greater than zero for all  $D > 0$ . It means that there is always a real solution to interest rate equation. Thus, there exists a market clearing interest rate for all levels of deposit demands,  $D$ .

If upper level of returns are lower than a critical level  $n^H < \hat{n}$ , threshold of deposits will be always positive,  $\hat{D} \geq 0$ . Figure 3.3 shows the graphical explanation of this condition. Hyperbole denotes  $\Delta$  and it intersects x axis at the threshold of deposits,  $D = \hat{D}$ . In this case,  $\Delta$  is greater than zero when deposit demand is lower than the threshold level,  $D < \hat{D}$ . It implies that if banks demand high level of deposits and their return level is not high enough, depositors won't lend their deposits.

## E.2 Solution

There exist two solutions for the participation constraint in equation (E.2). Figure 3.4 displays the graphical representation of the solutions. It shows that there are two roots and smaller one is the stable root. Analytical solution for the smaller root is given by:

$$r_{\text{part}}^d = \frac{(D+W)n^H - \sqrt{(D+W)^2(n^H)^2 - 4aD(D+W)(n^H - n^L)}}{2D} \quad (\text{E.4})$$

Bigger root can't be the solution of the problem, since it is unstable.



# Appendix F: Solution of Bank's Optimization Problem

## F.1 Interior Solution

Banks' profit function is concave when upper level of loan returns are smaller than the threshold level,  $n^H < 2 \left[ a + \sqrt{a(a-n^L)} \right]$ . Appendix E shows that when  $n^H < 2 \left[ a + \sqrt{a(a-n^L)} \right]$ , depositors will lend if banks' deposit demand is lower than the threshold level,  $D < \hat{D}$ . Therefore, solution of the banks' optimization problem will be an interior solution if deposit demand is lower than the threshold level,  $\hat{D}$ .

In order to solve bank's first order condition (3.7), derivative of interest rate with respect to deposits should be computed. Taking derivative of depositor participation constraint (E.2) with respect to  $D$  yields:

$$\frac{dr^d}{dD} = \frac{W(r^d)^2}{((D+W)n^H - 2r^dD)(D+W)} \quad (F.1)$$

Using equation (F.1), first order condition (3.7) can be rewritten as:

$$2D(r^d)^2 + (-2Wn^H - 3Dn^H) + (n^H)^2(D+W) = 0 \quad (F.2)$$

Depositor participation constraint (E.2) is rewritten as:

$$(r^d)^2 = \frac{D+W}{D} [n^H r^d - a(n^H - n^L)] \quad (F.3)$$

Combining equation (F.2) and (F.3) yields the deposit interest rates as a function of deposits:

$$r^d = \frac{D+W}{D} \left[ \frac{(n^H)^2 - 2a(n^H - n^L)}{n^H} \right] \quad (F.4)$$

In Appendix E, solution of depositor participation constraint given in equation (E.4) also defines the interest rates as function of deposits. Combining equation (E.4) and (F.4) gives the solution for the deposits:

$$D^{eq} = W \frac{(n^H)^2 (n^H + 1) - 4a(n^H - n^L)}{4a(n^H - n^L)(n^H + 1) - (n^H)^2 (n^H + 1)} \quad (F.5)$$

Since this optimum solution is always lower than the threshold level,  $D^{eq} < \widehat{D}$ , when  $n^H < 2 \left[ a + \sqrt{a(a - n^L)} \right]$ , optimum solution will be an interior solution

## F.2 Corner Solution

In the previous section, we prove that optimum solution is interior solution when  $n^H < 2 \left[ a + \sqrt{a(a - n^L)} \right]$ . But if the upper level of banks' loan returns are higher than the threshold level,  $n^H > 2 \left[ a + \sqrt{a(a - n^L)} \right]$ , banks profit function won't be concave and banks will demand all the money in the economy. In this case, banks' optimization problem has a corner solution and

depositors' participation constraint determines the solution. If banks returns are high enough for repayment, depositors can lend any available amount with appropriate interest rate, as shown in Appendix E. Depositors' endowment becomes the upper limit for the borrowing amount of banks. Replacing  $D=W^d$  in equation (E.2) yields the corner solution for the model:

$$r^d = \frac{(W^d + W)n^H - \sqrt{(W^d + W)^2(n^H)^2 - 4aW^d(W^d + W)(n^H - n^L)}}{2W^d} \quad (F.6)$$